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GROSS RENT MULTIPLIER RELIABILITY IN INCOME PROPERTY APPRAISAL

by

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A THESIS

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ABSTRACT

The central purpose of this study was to test the accuracy, and thus the reliability, of the gross rent multiplier as an appraisal device in the Edmonton area. Information, gathered in respect of apartment property sales which occurred from mid-1968 to the end of 1970, was analysed in order to determine (a) the statistical relationship between selling price (the dependent variable) and gross rent (the independent variable) and (b) the degree of predictive accuracy attached to application of the relationship in the appraisal of such income-producing real property. Tests consisted of simple linear regression and correlation analysis and comparison of predicted and actual values through application of regression equations, average multipliers, and median multipliers.

Secondary objectives of this study included assessment of contemporary attitudes in respect of the development and use of such appraisal devices and assessment of the effect, if any, of the November 9th, 1969, federal proposals for tax reform on general levels of value and on the predictive reliability of the gross rent multiplier.

The results of the study indicated that the gross rent multiplier cannot be considered satisfactorily reliable as an appraisal device in the Edmonton area. They also indicated that while pertinent and available literature may imply investor

reliance on the gross rent multiplier, they do not, because of the importance of other variables, demonstrate the logic of such reliance for anything but a preliminary guide to (or rough check on) the value of income-producing real property. Lastly, they indicated that the federal proposals for tax reform did not cause any recognizable change in apartment property values through 1970.

Of significance is the indication of an apparent lack of understanding in respect of the way(s) in which investors actually make their purchase decisions. This, together with the major findings of the study, can probably be taken as indication that further research in problems of income realty appraisal would be better devoted to examination of financial and behavioural characteristics of investors than to replication of this study in another urban area.

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CHAPTER I

INTRODUCTION

NATURE AND PURPOSE OF THE STUDY

The primary purpose of this study is to assess the reliability of the use of the gross rent multiplier in the appraisal of income properties.

The data on which this study is based are discussed in detail in Chapter III. The data are in respect of apartment property sales which occurred from July 1968 to December 1970 in the Edmonton area.

This study is designed to determine whether or not the gross rent multiplier (the ratio of capital value to gross income) may be considered to be reliable as an appraisal device in the Edmonton area. Secondly, it is hoped that comparison of the results with those obtained in previous studies will permit the drawing of some conclusions in respect of the degree of universality of the findings.

In one such previous study[1], twelve small samples of relatively homogeneous income properties were subjected to statistical analysis in order to measure the likely predictive

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[1]Richard U. Ratcliff, Current Practices in Income Property Appraisal--A Critique (Berkeley: University of California, 1967).

usefulness and accuracy of the regression equation to the practicing appraiser. In respect of 10 of the samples, which ranged in size from 12 to 25 properties, the author, Dr. Richard U. Ratcliff, noted that:

> ... the application of statistical tests showed that 80 to 96 percent of the variation in price was explained by the variation in income.[2]
>
> The statistical measures of confidence, the Student t and the F statistics were extremely high, giving assurance that the relationships underlying the equations were close to the measured relationships.[3]

In addition, Ratcliff applied the regression equation to each of the income figures for the items in each sample and compared the prices so determined with the actual sales prices as reported.

> In only one of the samples was the average deviation more than two percent of actual price, a degree of accuracy of prediction which is well within the usual limits of tolerance in appraisal.[4]

He observes that results of this sort, which are obtained from the study of a small number of properties such as would typically be known to, and considered by, the practising appraiser, should serve to encourage deeper exploration of such conversion ratios as value indicators. He indicates that careful stratification of similarly small samples could be

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[2]*Ibid.*, p. 45.

[3]*Ibid.*

[4]*Ibid.*

expected to produce even better results. He suggests that various classes of income properties might be tested for the linearity of the price-earnings relationship, and concludes by stating that:

> ... increasing efforts should be carried on among appraisers to secure more reliable data on relevant market behaviour and more comprehensive and penetrating studies in academic circles on the predictive reliability of such data.[5]

In a later study[6] which centers on 1964-1968 Greater Vancouver apartment sales data, the author, W.G. Farish, draws the following conclusions:

> The use of the traditional methods of selecting capitalization rates should be terminated as they do not result in market determined rates.[7]
>
> The findings with regard to gross income multipliers illustrate that they are capable of predicting values very accurately in many cases. Their use is to be encouraged where it can be shown that they are accurate.[8]

These conclusions run contrary to current appraisal theory which, for the most part, maintains that the traditional approach (i.e. capitalization of net income) is basic to the valuation of income-producing real property while the gross rent multiplier, which is used in appraisal to convert gross income to a value

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[5] Ibid., p. 52.

[6] W.G. Farish, "The Use of Direct Conversion Ratios and the Selection of Capitalization Rates in Residential Income Property Appraisal" (unpublished M.B.A. dissertation, University of British Columbia, 1968).

[7] Ibid., p. iii.

[8] Ibid.

estimate, is useful only as a rough guide or check.[9]

In drawing his conclusions in respect of the degree of usefulness of the gross rent multiplier, Farish notes, among other things, that the advantages of the gross rent multiplier are simplicity, data availability, and (with appropriate homogeneity) accuracy.[10] It is with these claimed advantages, particularly the latter, that the following pages are chiefly concerned. In order to facilitate particular comparison of the findings of this study with those of the Farish study, the analysis discussed in Chapter IV is the product of the same general techniques of statistical analysis applied to reasonably similar data. However, in comparing results, the reader should be aware of a number of differences in the two studies. One difference is that this study deals with data from a considerably smaller and younger urban area. As at June 30, 1971, 1,129,000 people resided in the Greater Vancouver area while the population of Metropolitan Edmonton totalled approximately 464,000.[11] In addition, the value of building permits issued in Metropolitan Vancouver in 1970 was just over $285 million while the value of building permits issued in Metropolitan

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[9]National Education Administration Committee, Real Estate Appraising in Canada (Winnipeg: Appraisal Institute of Canada, 1970), p. 255.

[10]Farish, "The Use of Direct Conversion Ratios and the Selection of Capitalization Rates in Residential Income Property Appraisal," p. iii.

[11]Financial Times of Canada, Industrial Locations in Canada (Montreal: Southam Press Ltd., June 14, 1971).

Edmonton totalled approximately $136 million.[12] As well, Vancouver is a "port" City with an economic base substantially different from that of Edmonton.

There are also some differences in basic data. The Vancouver study analyses the sale of 385 apartment properties, 94 of which are high-rise structures (including 42 frame-elevator buildings). By contrast, the Edmonton study centers on the sale of 157 apartment properties, all of which are walkup structures.[13] The average selling price of the apartment property sales considered in the Edmonton study is approximately $161,000 while the average selling price of the apartment property sales considered in the Vancouver study is $300,000.[14] Lastly, it should be observed that the Edmonton data are more recent (1968-70 vs. 1964-68), cover a shorter time span, and have been selected from a period of greater economic uncertainty.

Recognition of the latter prompted an extension of the study. Proposals for tax reform[15], announced in November of 1969 by the Hon. E.J. Benson, Minister of Finance, threatened to severely limit tax shelter benefits derived from ownership

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[12]Statistics Canada, "Building Permits, 1970," Catalogue 64-203.

[13]Attempts to secure gross income data in respect of the few recent sales of high-rise apartment properties in the Edmonton area were unsuccessful.

[14]Richard U. Ratcliff, "Don't Underrate the Gross Income Multiplier," Appraisal Institute Magazine, Vol. 14, Book 1 (1969-70), p. 19.

[15]Hon. E.J. Benson, Proposals for Tax Reform (Ottawa: Queen's Printer for Canada, 1969, Catalogue F32-169).

of income-producing realty. Imposition of a capital gains tax was also threatened. With respect to the former, the proposals for tax reform included three significant modifications to the existing depreciation, or capital cost allowance, system. Firstly, it was proposed that a taxpayer no longer be permitted to deduct from other income a loss created by capital cost allowance on the rental of real property. Secondly, it was proposed that each rental building costing $50,000 or more be placed in a separate capital cost allowance class, and on the sale of each such building the taxpayer bring into income any recaptured depreciation. Under the existing system, all buildings of a particular construction were pooled and, by periodically adding new buildings to the pool, it was possible for a taxpayer to postpone payment of tax on recaptured depreciation almost indefinitely. Thirdly, it was proposed that when depreciable property is bequeathed to someone other than a spouse, the beneficiary would for tax purposes inherit the tax cost of that property to the deceased. This would mean that the inheritor would start with the same base for depreciation as the deceased had when he died. Under the existing system, the inheritor of depreciable property was allowed to use the fair market value of the property as the base for depreciation, even though the estate was not required to pay any tax on any recaptured depreciation.[16] With respect to the taxation of capital gains, it was proposed that

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[16]In this case, the tax postponed became tax saved forever.

capital gains be subjected to a progressive tax as part of the general income tax system. Contrary to previous practice, depending on the nature of the asset, all or part of the gain would be included in income and taxed at the taxpayer's marginal rate.

The use of the gross rent multiplier can be expected to appropriately reflect current or future market value only if the same fundamental conditions existent in the past still prevail at the time for which predictions are being made. To the extent that a trend, or any fluctuation, occurred during the years considered, the general level of the gross rent multiplier (for some classes of real property, if not for all) might be changed, and it is clear that misleading results could occur in those cases where a gross rent multiplier is selected from data without recognition of material changes in fundamental conditions of the market. Since there is reason to believe that such a notable event as publication of the tax reform proposals did in fact contribute to the previously noted increased economic uncertainty, and can therefore be expected to have significantly affected investors' attitudes during the 1968-1970 period, a portion of the data and analysis in this study is devoted to separate study of those transactions which occurred "before" and those which occurred "after" publication of the proposals.

SIGNIFICANCE OF THE STUDY

Existing treatises which deal with the valuation of income-producing realty describe many methods or techniques, including the gross rent multiplier, which may be relied upon by investors. Some of these contend that the gross rent multiplier is not characteristically employed by investors in the market place and should therefore be relied upon by appraisers for only the roughest kinds of calculations prior to employment of more refined analyses; others intimate that the gross rent multiplier is a reliable appraisal device. The latter claim appears to be based on empirical evidence which indicates that the gross rent multiplier may be satisfactorily accurate, and that it is used with some degree of regularity by appraisers--a fact which, it is suggested, implies investor acceptance and use. This study presents additional evidence in respect of the reliability of the gross rent multiplier. It does not investigate the extent to which the gross rent multiplier is in fact used by appraisers or investors, but it includes, in the survey of pertinent and available literature, a search for some indication that it is or is not logical for the investor to use and rely upon the gross rent multiplier.

This study might be considered significant for other reasons. Firstly, whether or not the gross rent multiplier is considered to be a reliable appraisal device in the Edmonton area, the tests of Chapter IV may be expected to indicate the general

magnitude and nature of the effect (if any) of the tax reform proposals on the value of apartment properties. Secondly, the study may be expected to be of local interest. It consists of a collection and analysis of recent historic evidence in respect of 157 apartment property transfers which have occurred in the Edmonton area. This presents an amount of factual information, in one report, which has not likely been previously available to appraisers, brokers, and investors in this urban area. The inclusion, within the Appendix of this study, of 157 calculated gross rent multipliers (listed by size of apartment building) should be of particular interest to such persons. In addition, the application of simple regression analysis, via the computer subprogram, may be of interest to appraisers and others who are, or who are likely to become, concerned with computer applications in real estate investment analysis.

ORGANIZATION OF THE STUDY

This Chapter is an introduction to the study. Chapter II is devoted largely to the definition of terms used herein and to discussion of the development and use of direct conversion ratios. It deals particularly with the gross rent multiplier, and includes consideration of the support and criticisms given to this ratio and to its users. Chapter III outlines the data and methodology of this study. It includes both a discussion of the source, nature, and limitations of the raw data and a discussion of the statistical and other techniques of analysis employed herein.

Chapter IV presents the results of the analysis. Chapter V, the final Chapter, constitutes a summary of the study and contains conclusions drawn from the results of the analysis.

LIMITATIONS OF THE STUDY

A first limitation which must be acknowledged is the fact that this study does not test for evidence that the gross rent multiplier is in fact used by income-property investors. It simply relates gross income estimates to selling price data and measures the predicting accuracy of the resulting ratio.

In addition, it must be recognized that the study is limited to consideration of only one type of income-producing realty--the apartment property. Many other forms of income-producing realty (e.g. retail stores, offices, warehouses, manufacturing plants, and ranches) might have as easily been the basic subject matter of this study. However, there is reason to believe that a survey of recent sales of such properties would yield a relatively small sample (and one which is less satisfactory than that provided by apartment property sales) because such properties vary rather extensively in size and form. The desire for comparability with previous studies also dictated restriction to an apartment sample.

Other limitations include both the lack of availability of some raw data and related information which might have been meaningfully included in the study and the problems in interpreting some of the questionnaire responses.

CHAPTER II

GROSS RENT MULTIPLIERS

DIRECT CONVERSION RATIOS

The gross rent multiplier, which has been defined earlier as the ratio of capital value to gross income, is a direct conversion ratio.

Ratcliff, in a recently published research report, defines direct conversion ratios as:

> ... the ratios which appraisers use to convert income, usually current actual or economic rental income, to a capital figure representing market value of the property.[1]

In this same report, which is based on the study of actual appraisals of income-producing properties, he indicates that income is most often gross annual earnings, but states that it is also often found to be net income before depreciation, net income after depreciation, and cash flow.[2] Ratcliff also indicates that the value figure which the appraiser is most often called upon to provide to his client

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[1]Richard U. Ratcliff, Current Practices in Income Property Appraisal--A Critique (Berkeley: University of California, 1967), p. 39.

[2]*Ibid.*, p. 40.

(i.e. market value) is acceptably defined in the market as:

> ... the Most Probable Selling Price of the subject property if exposed to the market for a reasonable time.[3]

In addition to the gross rent multiplier, and other ratios which express the relationship between property income and the probable selling price of the property, direct conversion ratios encompass units of comparison which relate value to non-income characteristics of a property. For example, apartment properties may be valued through the use of such multipliers, or rules of thumb, as:

(a) sales price per suite.
(b) sales price per room.
(c) sales price per gross square foot of total building area.[4]

While these relate value to non-income characteristics, it can readily be seen that they all relate value to a measure of "size" or space, and space, of course, is one of the major features which draw income to the apartment property. Thus, although these relate value to non-income characteristics of the apartment property, they do relate value to a feature which, in turn, is related closely to a characteristic which is of significant interest to typical investors--i.e. rent-producing capacity. However, it is clear that while two apartment properties situated in a given neighbourhood may possess an identical number of suites, rooms, or square feet of building area, a significant difference in the size

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[3]Richard U. Ratcliff, Modern Real Estate Valuation (Madison: Democrat Press, 1965), p. 1.

[4]American Institute of Real Estate Appraisers, The Appraisal of Real Estate (Chicago: Author, 1967), p. 247.

and shape of the rooms, or in the functional layout of the suites, could show itself in a correspondingly significant difference in rent-producing capacity, a fact which would generally be reflected in a notable difference in the value of the two properties. Thus, one can argue that the best direct conversion ratio is that ratio which relates value most directly to the rent-producing capacity, and the most direct relationship is, of course, a ratio between value and "income" itself. Of such ratios, it is said that the gross rent multiplier is frequently the only comparative characteristic which can be applied.[5]

APPRAISER USE OF DIRECT CONVERSION RATIOS

Ratcliff notes that in his examination of 84 appraisals of income-producing real property, application of the gross rent multiplier was included in 25; of the remaining appraisals in which some form of direct conversion ratio was employed, the ratio was applied to various net income estimates. The latter are referred to by Ratcliff as price-earnings ratios. He indicates that in each of the appraisals which employed such direct conversion ratios, the ratio was derived, explicitly or implicitly, from actual sales of comparable properties for which selling price and income facts were available. In 19 of the appraisals, the direct conversion ratio was the only device employed to convert income to an estimate of capital value.[6]

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[5] _Ibid_., p. 351

[6] Ratcliff, _Current Practices in Income Property Appraisal--A Critique_, p. 40.

USE OF DIRECT CONVERSION RATIOS IN SECURITY ANALYSIS

Direct conversion ratios in real property appraisal have been likened to the price-earnings ratios which are often used by securities analysts in the evaluation of stocks. A widely accepted tenet is that a common stock is "worth" the present value of all future dividends.[7] However, calculation of such a figure demands both a projection of future dividends and selection of an appropriate discount rate, tasks which are generally admitted to be coarsely accomplished because of the many assumptions which must be made.

> Practicing security analysts generally overcome the latter problem by evaluating stocks in terms of price-earnings multiples rather than price-dividend multiples. The former problem usually is attacked by devising various rules of thumb for selecting an appropriate price-earnings ratio which can be applied to a company's existing level of earnings per share. The basis for these rules of thumb may range from the purely intuitive to elaborate statistical analysis.[8]

This is coincidentally the situation facing real property appraisers. The traditional income approach to the valuation of real property (i.e. capitalization of net income) is a present value calculation. It holds that the value of a property is the present "worth" of the net income it will produce during the remainder of its productive life.[9] Calculation of such a present value figure

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[7]Jerome B. Cohen and Edward D. Zinbarg, Investment Analysis and Portfolio Management (Homewood, Illinois: Richard D. Irwin, Inc., 1967), p. 222.

[8]Ibid., p. 239.

[9]American Institute of Real Estate Appraisers, The Appraisal of Real Estate, p. 228

demands the same income projection and discount (capitalization) rate selection, each of which is similarly based on a great many poorly-founded assumptions. Criticism of the traditional methods of capitalization rate selection touches on the latter. Then, just as some securities analysts stress the superiority of price-earnings ratios as a device for the valuation of stocks, some authors suggest the superiority of the gross rent multiplier as a valuation tool in the hands of the real property appraiser. The gross rent multiplier, which is the most common of the direct conversion ratios, is particularly likened to the price-earnings ratio[10], albeit to gross rather than net earnings.

The remaining pages of this Chapter are devoted to discussion of the development and use of the gross rent multiplier and to consideration of support and criticisms accorded the multiplier by the authors of available and pertinent literature.

THE GROSS RENT MULTIPLIER

The gross rent multiplier is calculated simply by dividing the selling price of a property by the gross annual rent which the property produced at the time of sale. Use of the gross rent multiplier implies the investigation of a group of fairly homogeneous properties in order to determine the relationship

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[10] W.G. Farish, "The Use of Direct Conversion Ratios and the Selection of Capitalization Rates in Residential Income Property Appraisal" (unpublished M.B.A. dissertation, University of British Columbia, 1968), p. 41.

between the gross income[11] and the selling prices for the purpose of selecting a typical or representative ratio which may be applied to the income of a similar property in order to estimate the most probable selling price of the similar property.

PROBLEMS OF DATA ACQUISITION

Problems are often encountered by the appraiser in his attempt to obtain market derived gross rent multipliers which show a suitable degree of consistency.[12]

The selling price, which consists of the total proceeds arising from the sale of the property, is sometimes a figure which can mislead the appraiser. For example, in a given sale the total proceeds may include a property which has been taken in trade, at an inflated price, as part, or all, of the down payment. It may be that the vendor receives $220,000 for his apartment property (which would normally be worth approximately $200,000 on the open market) because he agrees to accept the purchaser's acreage as having a value of $70,000 when he knows it is worth perhaps only $50,000. In another instance, the appraiser may be aware of the actual cash consideration given for a property, but he may not

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[11]Gross Income Multiplier (GIM) and Gross Rent Multiplier (GRM), in the context of this study, are one and the same.

[12]The appraiser generally gains knowledge of the selling price, the gross annual rent, and other details, by talking to one or both of the parties to the transaction, or by talking to other persons who have knowledge of the transaction.

know that the vendor had been under threat of foreclosure. On the other hand, a property may have been transferred by a retiring businessman to his son-in-law. In either of these latter examples, the selling price may be in unnatural relationship with the prevailing market. In other instances, the information given to the appraiser may be deliberately, or accidently, erroneous (i.e. not, in truth, the actual consideration given or received). An example of the latter may be in the case where the vendor assumes that the total proceeds arising from the sale of the property are equal to those dollars which are left after commissions, legal costs, and other selling expenses.

The gross rent, which is defined in this study[13] as gross yearly income (assuming full occupancy plus parking rental) as indicated by rentals at time of sale, can also cause the appraiser some difficulty. It may be an actual amount, or one which is estimated. If actual, is it the result of multiplying the rent receipts in the month of sale by 12, or is it the total of the receipts for the twelve months preceding the date of sale? If it is estimated, by whom has it been estimated, and on what basis? The vendor's estimate of the coming year's gross rents may be quite different from the purchaser's estimate, which in turn may be quite different from an estimate made by an involved real estate broker, or by an interested mortgage lender. The likelihood of different estimates from vendor and purchaser on the

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[13]See Appendix, page 136.

very day of sale is strong; the purchaser may be looking backward to his rather poor experience, while the vendor may be anticipating an immediate healthy increase in revenue because of his intention to place the property in the hands of an experienced and market-wise manager. Differences in management style will cause other income differences. An example of the latter may be in the case where one manager increases total revenue by selling electrical power to the tenants, by providing on-site parking for an added rental charge, and by installing coin-operated laundry equipment which he personally cares for, while another manager gives up the opportunity to earn such added income in favour of fewer management pressures. Lastly, despite the definition contained in the questionnaire, some respondents listed gross rent as an "after vacancy" amount, with some figures based on actual vacancy experience and some based on average vacancy experience. This latter tendency can probably be taken as a preference on the part of some owners, or managers, to think in terms of "effective" gross rather than the somewhat hypothetical gross defined in the questionnaire. The difficulty in considering effective gross is, of course, that vacancy and bad debts are too often estimated differently by different persons or, if such allowance is an actual amount, it is found to vary more with the quality of management than with market factors.[14]

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[14] Dr. Ratcliff alludes to the matter of inconsistency of gross income when he notes in his definition of direct conversion ratios that income is "... usually current actual or economic rental income ...". See page 11 of this study.

APPLICATION AND USE OF THE GROSS RENT MULTIPLIER

Authors of some textbooks and articles on the subject of real property appraisal go so far as to suggest that the gross rent multiplier is not really an acceptable appraisal method. It is stated that:

> ... as an index of value the gross income is useful, but only as a secondary device--a check on the result obtained by other and professionally recommended methods under the market, cost, and income approaches to value.[15]

> ... multipliers are often considered useful as rough measures of investment performance. Conversation with real estate brokers, investors, property managers, and appraisers ... indicated their general hesitancy to rely on such measures for any but the roughest kinds of calculations prior to more thorough investigation.[16]

> Other rules of thumb which are frequently employed by appraisers include primarily gross income multipliers, direct capitalization or multipliers of net income, and income and expense ratios. These rules can serve only as general guides for the purpose of assisting in formulating preliminary conclusions.[17]

> ... it is only capable of resulting in a rule-of-thumb estimate of value ...[18]

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[15]Alfred A. Ring, The Valuation of Real Estate (Englewood Cliffs, N.J.: Prentice-Hall, Inc., 1970), p. 145.

[16]Fred E. Case, Los Angeles Real Estate: A Study of Investment Experience (Los Angeles: Real Estate Research Program, University of California, 1960), p. 47.

[17]S. Edwin Kazdin, "How to Use Income Data in the Appraisal of Apartment Property," The Appraisal Journal, XXVII, No. 3 (1959), p. 392.

[18]National Education Administration Committee, Real Estate Appraising in Canada (Winnipeg: Appraisal Institute of Canada, 1970), p. 255.

This tendency of textbooks, and other writings, to discard the gross rent multiplier by brief comments on its usefulness for first approximations of value and warnings as to its unreliability, was noted by Ratcliff.[19] This unreliability, he suggests, is said to arise in large part from the differences which exist among properties in their operating ratios (ratio of operating expenses to revenues).[20] Traditional appraisal theory and practice is based on the belief that the first and foremost concern of the investor is for net income, and it is maintained that the investor would care little for a property which boasts a high gross income if operating expenses are so great that little, or no, net income remains. In other words, use of the gross rent multiplier may provide misleading indications of value because the multiplier makes no provision for differences in net incomes when gross incomes may be similar.[21] The appraiser is exhorted to use the gross rent multiplier with caution because:

> ... the multiplier converts into value gross rather than net income. It is entirely possible that a property which produces a comparable gross income may yield inadequate or even no net income because of excessive operating or maintenance cost due to faulty construction or inequitable contractual commitments written into long-term lease agreements.[22]

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[19]Richard U. Ratcliff, "Don't Underrate the Gross Income Multiplier," Appraisal Institute Magazine, Vol. 14, Book 1 (1969-70), p. 266.

[20]Ibid.

[21]National Education Administration Committee, Real Estate Appraising in Canada, p. 255.

[22]Ring, The Valuation of Real Estate, p. 146.

> ... the use of the multiplier assumes uniformity among properties in their operating ratios.[23]

> ... ignores the effect of variations in vacancy losses ...[24]

The gross rent multiplier is also said to be unreliable because it:

> ... disregards the influence of mortgage financing.[25]

and because:

> ... consideration of remaining economic life appears entirely ignored.[26]

In addition, the gross rent multiplier is also said to be used inflexibly.[27] The suggestion is that once a multiplier is adopted, there is a tendency for users to regard it as built-in to the market, never to change.

> There are so many seven times (7x) buyers and sellers ...[28]

Some authors indicate, however, that the gross rent multiplier has merit as an appraisal device, and most of these agree that it is a simple device which is derived from readily

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[23]Ibid., p. 147.

[24]Paul F. Wendt and Alan R. Cerf, Real Estate Investment Analysis and Taxation (New York: McGraw-Hill, 1969), p. 186.

[25]Ibid.

[26]Ring, The Valuation of Real Estate, p. 147.

[27]Farish, "The Use of Direct Conversion Ratios and the Selection of Capitalization Rates in Residential Income Property Appraisal," p. 50.

[28]Leonard H. Scane, "The Mystic Seven," The Appraisal Journal, XXVI, No. 3 (1958), p. 390.

available data and which is easily applied by the appraiser.

> The advantage of this type of multiplier is that it is easy and quick to apply.[29]

> Probably the greatest contributing factor for its continued use is that gross rentals are more easily ascertained and easier to acquire from the owner of a property than the net income. In addition to this, the gross capitalization process is much more simple to apply (multiplying) than the methods generally advocated for the capitalization of net income (where one must divide and then multiply). Therefore, its use will also have the advantage of mathematical expediency over the capitalization of net income.[30]

However, while some authors consider the gross rent multiplier to be an appraisal technique or method which can stand on its own as a device able to yield a reliable final estimate of the market value of income realty, some, by attention to other uses of the multiplier, could well be indicating their rejection of it as a means of obtaining a final value estimate.

> It is well known that in the trade, among real estate brokers, mortgage lenders and appraisers, the direct conversion ratios are the basis for at least first approximations of value for some classes of property in some markets.[31]

> ... multipliers are often considered useful as rough measures of investment performance.[32]

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[29]National Education Administration Committee, Real Estate Appraising in Canada, p. 255.

[30]John J. Carney, "The Development and Use of Gross Income Multipliers," The Appraisal Journal, XXVI, No. 2 (1963), p. 226.

[31]Ratcliff, Current Practices in Income Property Appraisal--A Critique, p. 42.

[32]Case, Los Angeles Real Estate: A Study of Investment Experience, p. 47.

> In many areas, lay investors use the multiplier as a cardinal guide in judging the quality of property purchase offers.[33]
>
> Multipliers are useful as general guides in testing comparability in the market data approach.[34]
>
> In the absence of expense figures, it is frequently possible to obtain a reliable estimate of the market interest rate from a gross capitalization rate, or the reciprocal thereof, which is an annual gross rent multiple.[35]
>
> It will provide him (the assessor) with a warning signal in the event any action should be taken, relative to equalization within a class or equalization among the various classes of property.[36]
>
> It serves as a rational check and balance on the indicated value as a result of the cost approach and the market approach.[37]
>
> ... it will serve as a measurement of all kinds of depreciation ...[38]
>
> ... is a very welcome tool in arriving at an indicated value in an old run-down neighbourhood where sales have disappeared as a value index but where rents are stabilized.[39]

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[33]Ring, The Valuation of Real Estate, p. 145.

[34]American Institute of Real Estate Appraisers, The Appraisal of Real Estate, p. 337.

[35]Ibid., p. 275

[36]Carney, "The Development and Use of Gross Income Multipliers," p. 227.

[37]Ibid., p. 226.

[38]Ibid.

[39]Ibid.

> ... should prove very useful during those periods of the business cycle when there is very little or no market for residential property.[40]

> ... may also be used on some occasions for a specific appraisal, in the following manner:
>
> 1. To establish a value in a preliminary survey of any type of property.
> 2. To establish an indicated value for the income approach in the appraisal of residential and apartment property.
> 3. To provide a check or an indication of the value in correlation of the three approaches.[41]

NEED FOR STRATIFICATION OF DATA

The writings of those authors who support use of the gross rent multiplier, either as a means of obtaining a reliable final estimate of market value or for other purposes, suggest that its faults may be at least partially overcome via selectivity or stratification of the available market data.

> ... the appraiser must first make certain that the property from which he will derive the multiplier is comparable to the subject property.[42]

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[40] _Ibid._

[41] _Ibid._

[42] Carney, "The Development and the Use of Gross Income Multipliers," p. 222.

> ... care must be taken that the properties have similar characteristics before the multipliers can be used.[43]

> The gross income multiplier should be used only with great care and will give satisfactory results where there is a high degree of comparability of properties, locations, and market conditions.[44]

> They must be used with great care and only under highly comparable property conditions as to age, size, operating ratio, etc.[45]

> If the subject property is an income property, the comparables must be similar types of properties with similar operating expense ratios and remaining economic lives.[46]

One author suggests that comparables should be divided into "classes" according to such criteria as date of sale, quality of construction and workmanship, construction materials, size of building (height, rooms, etc.), age, and locality.[47] Then, the

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[43]National Education Administration Committee, Real Estate Appraising in Canada, p. 254.

[44]Kazdin, "How to Use Income Data in the Appraisal of Apartment Property," p. 392.

[45]American Institute of Real Estate Appraisers, The Appraisal of Real Estate, p. 337.

[46]National Education Administration Committee, Real Estate Appraising in Canada, p. 255.

[47]Carney, "The Development and Use of Gross Income Multipliers," p. 223.

suitability of comparables may be judged by examining:

1. Terms of sale (land contract, cash, or etc.).
2. Type of conveyance (contract, deed, or etc.).
3. Conditions surrounding sale.
4. Motives of buyer and seller.[48]

With the discard of unsuitable comparables, the remaining properties within each class may be utilized to provide multipliers for various types of property.

Clearly, many theorists are concerned because the multiplier, by its simple nature and history of use, appears unlikely to consider all those factors which seem relevant in the selection of comparable income properties. It is feared that indications of value obtained through use of the gross rent multiplier will be misleading because the multiplier fails to adequately reflect varying degrees of risk among and between properties. It is said to reflect differences in volume of income but not differences in quality and durability of income.

> ... the multiplier will vary widely for different types, areas, classes, or grades of property, as well as with the type of neighbourhood, depending on the amount of risk involved in the investment.[49]

> Although sales in the real estate market recommend themselves as prime evidence of values, the non-homogeneity of real estate, variations in market activity and performance, differences in terms and conditions of sale, and questions of comparability add up to a formidable group of problems ...[50]

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[48] _Ibid._

[49] _Ibid._, p. 225

[50] Paul F. Wendt, _Real Estate Appraisal_ (New York: Henry Holt and Company, 1956), p. 267.

Formidable though the problems may be, the general suggestion is that an acceptable multiplier may be derived from a sample of satisfactorily homogeneous properties. Opinions in respect of satisfactory comparability have been noted. Ratcliff has suggested some basic measures of comparability or homogeneity--size, type, construction, age and location. Additionally suggested measures include--similar quality of construction and workmanship, similar quality of tenants, similar lease conditions and terms, similar level of tenant services, similar terms of sale (i.e. financing), and similarity in date of sale. There would appear, in fact, to be an almost unlimited number of measures of comparability to consider if this were desired. The problem which would seem to be inherent in the use of a great number of such measures is that too few properties would be found suited to use in the appraisal of a given property. Clearly, the appraiser's hope is that a group of closely comparable properties will yield gross rent multipliers which fall in a narrow range such that they show a clear central tendency, or mode. It may be suspected that gross rent multipliers drawn from very small samples will tend not to be strongly clustered.

> If there were a sufficient number of rented or sold properties from which to establish a sound gross multiplier, and if there were a sufficient number of rented properties comparable so that subject's economic rent may be estimated with validity, a very strong indication of value can result.[51]

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[51] Neville F. Allison, "Fundamental Appraisal Thinking," The Appraisal Journal, XXXII, No. 4 (1964), p. 583.

The desire to ensure that the finally selected multiplier comes from a large sample of multipliers has led several authors to suggest that it would be wiser to adjust some comparables for differences with the subject (i.e. for differences with the property to be valued) rather than delete them from the sample.

> ... the appraiser should guard against the generalized use of these techniques (rule-of-thumb value indicators) without adjustment for differences in real estate taxes, plan and design, physical condition, location, and character of tenancy.[52]

> ... make adjustments to the sale price for differences in the facts or conditions of the comparable properties with respect to any subject properties.[53]

> Prices are commonly "adjusted" to compensate for differences between the property sold and the property appraised. But the appraiser who hopes to adjust recent sales must reduce items of comparison to a meaningful number. It is doubtful if most buyers would weigh very heavily more than three or four items in negotiating a sale. If the appraiser considers more than a few items, he can confuse the method to the point where the logic of his estimate may be hidden. Furthermore, if the sale needs more than three or four items of adjustment, probably it is not very comparable and may deserve rejection, not adjustment.[54]

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[52] Kazdin, "How to Use Income Data in the Appraisal of Apartment Property," p. 392.

[53] Carney, "The Development and Use of Gross Income Multipliers," p. 223.

[54] William M. Shenkel, "Modernizing the Market Data Approach," The Appraisal Journal, XXXV, No. 2 (1967), p. 190.

> If we concentrate on the more common items, we would probably find that on improved property the land area, the building condition, the floor area, and building features would lead most other adjustment factors. We are also inclined to weigh relative advantages of location ... physical features ... time of sale ...[55]

These suggestions are countered by other authors who claim that adjustments are intellectually dishonest and/or that adjustments are not required because a sample need not be large in order to yield a reliable multiplier.

Farish notes that a prime attraction of the gross rent multiplier as a valuation tool is its simplicity.

> Detailed consideration of numerous factors is seemingly at odds with this ... it is possible to criticize attempts to add sophistication to something that is considered to be a simple valuation device ...[56]

Just as it is claimed that some, or perhaps all, of the traditional methods of capitalization rate selection fail to meet the test of market determination, so can it be said that selection of a gross rent multiplier from adjusted data will reflect the appraiser's subjective evaluations to the extent that the resulting multiplier will relate weakly (if at all) to the market.

Other authors have commented as follows:

> Adjustments are never made to either the income, nor the price, to change this multiplier.[57]

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[55] Ibid., p. 191.

[56] Farish, "The Use of Direct Conversion Ratios and the Selection of Capitalization Rates in Residential Income Property Appraisal," p. 57.

[57] National Education Administration Committee, Real Estate Appraising in Canada, p. 254.

> ... care must be taken not to adjust the gross income, nor the "raw" market prices ...[58]

Another author has indicated that the objective of multiplier analysis is defeated when the appraiser adjusts multipliers and their supporting data to produce a multiplier which is believed typical of the market. In addition to claiming that the result can only be based on some preconception[59], he says:

> It is only when sales do not meet minimum standards of comparability that appraisers must study real estate trends or "adjust" sales.[60]

This raises the question of minimum standards of comparability, a question which the investigated writings do not appear to answer in any direct fashion. The textbook of the American Institute of Appraisers, in reference to the gross rent multiplier, says simply that:

> Sufficient accurate data must be gathered on comparable properties which have changed hands in normal transactions at or near the time of appraisal, before this process can be used with confidence.[61]

Ratcliff discusses this matter in similarly general terms:

> ... the appraiser may utilize it with confidence over the short run provided that it is based on a sample of reasonable size comprised of reasonably comparable properties.[62]

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[58]Ring, The Valuation of Real Estate, p. 147.

[59]W.M. Shenkel, "Characteristics of Gross Income Multipliers," The Real Estate Appraiser, XXXIV (January-February, 1968), p. 26.

[60]Shenkel, "Modernizing the Market Data Approach," p. 187.

[61]American Institute of Real Estate Appraisers, The Appraisal of Real Estate, p. 337.

[62]Ratcliff, "Don't Underrate the Gross Income Multiplier," p. 271.

Reference to one of Ratcliff's earlier works[63] sheds considerable light on his concept of reasonableness. In the previously noted study of 84 appraisal reports, Ratcliff examined, via regression analysis, several small groups of income-producing properties. He claims to have been in possession of only the barest of details in respect of reasonably homogeneous properties contained in samples which ranged in size from 12 to 25. Each of these samples was made to produce simple linear regression equations. Results of significance tests showed good relationship between the current potential income (100% occupancy at current rental rates) and the selling price. One of the conclusions drawn from the results of the comparison of predicted prices with actual prices reads as follows:

> ... if an appraiser had been required to appraise all of the properties included in all of the samples and if he had used only the reported current incomes and the direct conversion ratios derived through using the regression equation representing the basic relationship in the appropriate sample, his value answer would have been within two percent of what the property actually sold for on the average.[64]

Clearly, there is some difference of opinion on the matter of suitability of samples--with some authors calling for large samples (possibly with adjustment of differences to ensure same), and other authors decrying adjustments (many stating their belief

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[63]Ratcliff, Current Practices in Income Property Appraisal--A Critique.

[64]Ibid., p. 46.

that relatively small, but reasonably homogeneous, samples are satisfactory). There is good agreement, however, in respect of the need for the user of the gross rent multiplier to ensure that it is selected from a satisfactorily homogeneous sample of properties which are highly comparable to the property under appraisal. The limitation of this study to data in respect of one form of income-producing realty--the apartment property--is in recognition of this claimed need for some measure of homogeneity in the sample of properties from which the gross rent multiplier is to be selected. Testing of the reliability of the multiplier by way of stratification of the properties into more and more comparable groupings is further such acknowledgement.

RELIABILITY OVER TIME

To the extent that the gross rent multiplier is, and will continue to be, used, it would appear desirable to note the attitude of some authors in respect of its reliability over time. Ratcliff indicates the following:

> To be useful, the sample ... should be constantly improved in quality by replacing the older cases with newer sales and increasing the size of the sample as more appropriate data become available.[65]

> ... the accepted conversion ratios change with changes in exogenous market forces and with investors' attitudes and expectations. But

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[65]Ratcliff, Current Practices in Income Property Appraisal--A Critique, p. 47.

> these changes are incorporated slowly into the market process and there is a lag in the adjustment of the accepted ratios to the new conditions.[66]

> Though the GIM ratio may change over time, and will vary among different markets and properties with different productivity characteristics, the appraiser may utilize it with confidence over the short run ...[67]

These comments imply a relatively slow change of gross rent multipliers over time and a need for routine up-dating of market data to ensure ongoing reliability of the multipliers. Another author offers comments which are somewhat to the contrary.

> If any conclusion is warranted from the records available, it is that capitalization of current gross or net earnings, or of those for only one or two past years, is a very poor basis for an investment decision.[68]

> ... the year to year changes in the gross income-acquisition price ratios indicate that although an investor may use the multiplier to estimate an initial purchase price and anticipated return, he may never except in the year of purchase enjoy this return.[69]

Again, there is disagreement. If expenses and income change markedly from year to year, yet the gross rent multiplier can be expected to change slowly over time, then it must be true that property values fluctuate considerably from year to year. Considered from another viewpoint, when income is relatively

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[66]Ratcliff, "Don't Underrate the Gross Income Multiplier," p. 270.

[67]Ibid., p. 271.

[68]Case, Los Angeles Real Estate: A Study of Investment Experience, p. 80.

[69]Ibid., p. 47.

unstable, property values will show some constancy over time only if the multiplier is somewhat erratic and therefore of low reliability as an appraisal device.

Neither observation, of itself, can be considered to be descriptive of the market within which income-producing realty is traded, and available and pertinent literature appears to indicate that such disagreement could not be resolved in any event. This indication is based on arguments which suggest that the investor is not as concerned with annual income as some authors intimate.

BENEFITS OTHER THAN INCOME

One author states that direct conversion ratios, and similar predictive devices, are likely a good measure of the relationship between actual sales and actual rents but are not necessarily an explanatory thing.

> ... the results of a regression only express an association of independent with dependent variables, and do not necessarily establish causal relationships.[70]

Another author notes that price is sensitive to and reflects many benefits arising from ownership.[71] Price, notes Dr. Herbert B. Dorau, almost always reflects the purchaser's opinion of the worth, at

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[70]Gene Dilmore, "Multiple Regression Analysis as an Approach to Value," Appraisal Institute Magazine, Vol. 15, Book 2 (1971), p. 54.

[71]Herbert B. Dorau, "The Capitalization Rate: Mirage or Will-o'-the-Wisp?," The Appraisal Journal, XXIX, No. 1 (1961).

a moment in time, of anticipated economic advantages other than, or in addition to, net income--e.g. land appreciation, equity build-up resulting from mortgage principal reduction, tax shelter, protection (hedge) against inflation, and pride and prestige of ownership.

> Why then did we ever coddle to the idea that price was uniquely or even exclusively related to the indicated dollar income?[72]

Dorau explains the connection between price and income by noting that present income has traditionally been, and still appears to be, valued and important. While observing that we have come to recognize that it is, in fact, future expected income which is more important, we tend to stay with present income because it is more calculable and not so much the product of subjective opinion of diverse persons.[73]

> This is hardly an excuse for failure to recognize all other sources and forms of gains; particularly when, as in our present political economy, these other valued gains are so substantial and even at times dominant in determining price.[74]

Dorau is not disputing the usefulness of such direct conversion ratios as the gross rent multiplier, he is merely putting them in proper perspective.

> Now we know, and should begin to act with intelligence on the basis of our understanding, that price is the product of many expectations and not uniquely or exclusively

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[72] _Ibid._, p. 22.

[73] _Ibid._

[74] _Ibid._, p. 23.

> related to the current income flow or even a projection of that income flow and that earnings or income price ratios are not capitalization rates.[75]

In other words, the gross rent multiplier, or its reciprocal, is not the rate per cent at which all the economic gains for which the price was paid were discounted to produce the price.[76] The ratio by which such gains and the price is related is no more correct than anyone's estimate of all future income and other economic gains. Only the price is right.[77]

Others have commented, indirectly, on this same matter.

> It (the GIM) is probably the most important market fact of all market facts for it expresses in a single ratio the final result of all of the multifarious price-establishing factors; the productivity characteristics of the property; the expectations and investment objectives of competing buyers and sellers; the conditions of demand and supply within the real estate market and the external factors which influence market activity and market price determination.[78]

> The fundamental principle underlying the use of a gross multiplier is that actual sales are related to actual rents, and this relationship, therefore, can be translated into a similar relationship with regard to a subject property. Thus, it would make no difference whether, by a residual technique a property reflects an annual loss or a net profit, the relationship between actual sales and actual rents is still reflected by the market.[79]

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[75] Ibid.

[76] Ibid., p. 29.

[77] Ibid., p. 28.

[78] Ratcliff, "Don't Underrate the Gross Income Multiplier," p. 265.

[79] Allison, "Fundamental Appraisal Thinking," p. 583.

However, while the former may suggest that many factors other than magnitude of income are acting to influence value, and while the latter may be interpreted as indicating that a property can have value despite failure to produce a positive net income, they do not acknowledge the fact that the gross rent multiplier, or any other direct conversion ratio, may not logically be expected to explain the investor's capitalization of ownership benefits, and may not therefore show a sufficient degree of reliability as an appraisal device.

Dorau has indicated that there are many benefits to be derived from ownership of income realty other than the receipt of annual income. He has also indicated that some of these other gains will at times be so substantial as to become dominant in determining prices. Investors' expectations in respect of appreciation in land value is noted as a possible influence on price. Appreciation potential of this sort was undoubtedly important to investors in the Edmonton area during the 1968-1970 period. However, because some apartment property purchasers would suffer tax liability from such increases in value while other purchasers were permitted to treat such an increase as a tax-free capital gain, this potential gain would have been more appealing to some buyers than others and could therefore have been expected to cause some buyers to be willing to pay more for a given apartment property (all other things equal) than would other buyers. Moreover, chiefly because of locational attributes, the

land beneath some apartment buildings would undoubtedly have been expected to appreciate in value more notably than the land beneath other apartment buildings and, dependent upon investor expectations in this regard, prices paid should be found to vary (again, all other things equal).

Investors' expectations with respect to equity buildup which may result from reduction of mortgage principal is also noted as having significant influence on price. The rate, and thus the attractiveness, of equity buildup is a function of mortgage characteristics (i.e. loan-to-value ratio, interest rate, and amortization term)--thus, since existing or available mortgages may vary considerably from apartment property to apartment property, investors can be expected to show a tendency to pay different amounts for differently financed properties (all other things equal).

Tax shelter is listed as another important investor expectation. This benefit, or gain, arises from the transfer of loss (suffered by an apartment because of fully applied capital cost allowance) to the owner's "other" taxable income. Since the benefit of such loss transfer is dependent upon the tax bracket occupied by the owner it will have been prized differently by different owners. Thus, a given apartment property which was purchased in part for tax shelter benefit can be expected to have had a different value in the eyes of different potential purchasers.

Similarly, varying expectations in respect of apartment properties' ability to match general levels of inflation, and thus serve as some protection against erosion of the purchasing power of the dollar, would likely have been reflected in varying purchase prices.

Lastly, the fact that some owners have no emotional ties to their apartment properties while others (possibly occupants of one of their own suites) show considerable pride of ownership, may also be reason to suspect a variation in prices paid (all other things equal).

Despite such commentary and reasoning, there has been an ongoing quest for a type of ratio which will serve as a reliable predictor of values across a wide spectrum of income-producing real property. Ratcliff says:

> It is a hypothesis worthy of testing that these (direct conversion) ratios become built-in market guides to investors who are buying or selling this kind of property and to lenders who are considering the advance of capital on mortgage security. Thus, the ratios become powerful factors in market price establishment, a fact which imparts predictive value to these same ratios.[80]
>
> It is a further hypothesis, subject to intensive research check, that the apparent stability in the relationship of income and value as expressed in the Gross Income Multiplier or Price-earnings Ratio is based on a widespread acceptance of given ratios in local real estate markets by brokers, appraisers, buyers, sellers and lenders as

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[80] Ratcliff, Current Practices in Income Property Appraisal--A Critique, p. 42.

> rules of thumb for arriving at a first approximation of the value of various income property types. To the extent that this is true, an accepted ratio becomes a price-determining factor in the price establishment process since all parties to each transaction tend to employ the same or nearly the same ratio in arriving at their first approximations of the price which they might be willing to accept, the bid which they might be willing to make or the basis for the loan which they consider to be sound in relationship to the value of the collateral property. If this generally accepted ratio becomes well established and a major factor in price establishment, it follows that a small sample of actual sales, properly processed as the basis for a regression equation, will provide a highly dependable basis for predicting the most probable selling price for a comparable property.[81]

> Farish says:

> ... there is some evidence and recognition of them (direct conversion ratios) becoming "built-into" the market for certain classes of property. That is, as investors and brokers use such ratios with some regularity, transactions in the market place begin to reflect these relationships.[82]

> To the extent that the foregoing is true, it might seem ... reasonable to attempt to resolve whether direct conversion ratios can be of any primary value by theoretical analysis and empirical testing.[83]

Later, within the pages of the same works, Ratcliff and Farish make the following comments:

> There is little question but that participants in the real estate market rely extensively on direct conversion ratios in investment decision-

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[81] *Ibid.*, p. 51.

[82] Farish, "The Use of Direct Conversion Ratios and Selection of Capitalization Rates in Residential Income Property Appraisal," p. 2.

[83] *Ibid.*

> making. This hypothesis should be tested in connection with a comprehensive study of investor behavior and his investment calculus which could provide a sounder basis for the appraiser's predictions.[84]
>
> ... the results would seem to support the theory that because market participants actually use multipliers they become built into the market. What is really the concern is the best prediction, not the method used. To the extent that the average multiplier results in the best prediction only confirms that this is the way market participants actually predict values.[85]

Neither of these authors offer any evidence which points to widespread use of the gross rent multiplier by investors. Ratcliff presents evidence which is claimed to point to general use of direct conversion ratios by appraisers.[86] He also presents evidence which is claimed to indicate that direct conversion ratios may be expected to show an acceptable degree of reliability as value predicting devices. Farish, after further, and rather more extensive, study of the strength of relationship between gross income and the selling price of apartment properties, supports

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[84]Ratcliff, Current Practices in Income Property Appraisal--A Critique, p. 52.

[85]Farish, "The Use of Direct Conversion Ratios and the Selection of Capitalization Rates in Residential Income Property Appraisal," p. 127.

[86]It is interesting to note the following comment made by Dr. Paul F. Wendt, University of California, Berkeley, in an article published in September 1970 by the Canadian Association of Real Estate Boards ("Recent Development in Appraisal Theory," The Canadian Realtor, Volume 16, No. 3):

> "Empirical evidence showing the prevalance in the use of the gross multiplier techniques by appraisers is not wholly satisfactory."

the "reliability" claim.[87] It would appear that both men are led by enthusiasm for the apparently positive nature of their regression (and related) analyses to the point where it seems natural to infer that the gross rent multiplier is indeed a favored and widely used tool of the investor.

In a later article (one which is based largely on the findings of the Farish study) Ratcliff makes the following statements:

> The GIM is widely used by appraisers, assessors, mortgage lenders and real estate brokers to estimate market value for their various purposes.[88]
>
> The GIM is freely used by all participants throughout the real estate market (buyers, sellers, brokers, agents, salesmen, lenders, developers, assessors, and appraisers) in relationship with various forms of decision making.[89]
>
> In summation, the GIM, probably because of its simplicity and readily available information on the two components, income and price, is widely employed in the decision-making of all market participants and thus becomes built-in to the price establishment process.[90]

Again, no evidence is offered which would indicate widespread investor use of the gross rent multiplier; claims in respect of investor use appear to be based solely on the apparent predictive accuracy of the gross rent multiplier.

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[87]Results discussed in Chapter IV of this study provoke some question of the validity of conclusions presented by Farish in respect of his tests of multiplier accuracy.

[88]Ratcliff, "Don't Underrate the Gross Income Multiplier," p. 264.

[89]*Ibid.*, p. 270.

[90]*Ibid.*, p. 271.

It would seem reasonable to conclude that the foregoing survey of pertinent and available literature suggests that appraisers, and others who are interested in patterns of real estate activity and in general forecasting of prices, have a useful tool in the gross rent multiplier[91], but that it would be illogical to expect that investors rely on a simple ratio such as the gross rent multiplier to make their final decisions (i.e. the decision to buy and the decision to pay a certain price). Not only does there appear to be no evidence to show that the gross rent multiplier is used to any significant degree by investors, but there is evidently a serious lack of understanding in respect of the ways in which investors actually make their decisions to buy and to pay a certain price. What appears necessary is a study which might take the form of an enquiry devoted to discovery of major variations in the financial and behavioural characteristics of investors. Perhaps this will show that the appraiser should be concerned more with stratification and analysis of the investor than with stratification and analysis of the income-producing realty.

This Chapter has centered on discussion of direct conversion ratios with emphasis on the development and use of the gross rent

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[91]Subject, generally, to the qualification that users (a) obtain the multiplier from homogeneous samples which are of reasonable size and which are constantly updated and (b) apply the multiplier to properties which are highly comparable to those which make up the sample.

multiplier. Some definition and description of direct conversion ratios, as well as some indication of their use by appraisers and security analysts, has been drawn from a survey of available and pertinent literature. Various authors' attitudes in respect of the usefulness of the gross rent multiplier have been examined. Comments about problems of data acquisition have been outlined, and remarks in respect of the application and use of such multipliers have been noted. The latter were seen to evidence general agreement that notwithstanding the nature of use of the multiplier, or limitations which may exist in its application, data stratification is essential. There was also indication that passage of time demands updating of the data. Most importantly, however, some of the comments indicated that non-(annual)income benefits flow to the owners of most, or all, income-producing realty, and that whether or not these are readily quantifiable they are sometimes so substantial as to become dominant in determining prices. Thus, there may be good reason to suspect that appropriate testing of the gross rent multiplier will indicate that it is not, of itself, a satisfactory appraisal device.

Chapters III and IV are devoted largely to discussion and analysis of data in an independent test of the accuracy (and thus the reliability) of the gross rent multiplier in the appraisal of apartment properties in the Edmonton area.

CHAPTER III

DATA AND METHODOLOGY

DATA

Bi-monthly reports of real property transfers, distributed by Teela Market Surveys, were examined for data concerning the sale of 386 apartment properties in the City of Edmonton for which transfers were registered in the Land Titles Office between July 1, 1968, and December 31, 1970.

Apartment properties sold by Agreement for Sale are not included in the Teela Market Surveys data; neither are those apartment sales in which, for one reason or another, registration of the transfer is postponed. However, since very few apartment property sales are by agreement, and postponement of transfer registration is rare, perhaps no more than three or four per cent of apartment sales will be overlooked through reliance on the Teela Market Surveys data.

The July 1, 1968 to December 31, 1970 time interval was chosen for two major reasons. Firstly, the examination of sales took place mainly in February and March, 1971. This suggested the desirability of choosing the 1970 year-end as a cut-off date for sampling. Secondly, since economic conditions change rather rapidly over time, it was felt that data should be limited as

much as possible to that which could be considered recent. Three hundred and eighty-six transfers were deemed to provide a rather significant initial data sample when one considers that this figure, relatively large in itself, probably represents over 95 per cent of all sales which took place during the above time interval. Another reason for selection of the mid-1968 commencement date was to allow for study of the effects of the tax reform proposals. For this purpose, it was desirable to procure a sample which was nearly equally divided between sales which took place before the announcement of the proposals and those which occurred after the announcement.

The Teela Market Surveys' information did not reveal the vendors' and purchasers' addresses, nor did it indicate the apartment buildings' age and construction. This information was obtained from the City of Edmonton Assessor's Department. The age and construction of each apartment building was required in order to further judge the suitability of the sample, and the questionnaire could not be mailed until the vendors' and purchasers' addresses were determined. Examination of the roll card and building card, for each of the 386 properties, provided the necessary data. An estimate of the number of rooms per building was also obtained at this time. In the end, for each of the 386 apartment properties, the writer possessed the following data:

1. Municipal address
2. Legal description

3. Assurance Fund Value (assumed to be the selling price)
4. Basic construction
5. Age of building (at date of sale)
6. Registration Date (assumed to be the date of sale)
7. Number of suites
8. Number of rooms (including baths)
9. Vendor's name and address
10. Purchaser's name and address
11. Land and building assessment.

Questionnaires were not mailed in respect of all properties in the sample. Thirteen of the sales were found to be of apartment buildings which were more than 20 years old at the date of sale. These were considered to be a small and relatively non-comparable group of properties which would have destroyed some of the homogeneity of the larger group, and they were therefore deleted from the sample. In addition, 22 other properties were found to be unusual in some fashion or other (e.g. some were private dwellings converted to apartment properties, and some could more logically be called rooming houses rather than apartment buildings) and these, too, were deleted from the sample.

The remaining 351 transfers were assumed to be valid sales, and the questionnaire which is shown in the Appendix, page 136 was mailed to each known vendor and purchaser.

As has been noted, the questionnaire requested a reply in respect of gross income only. Owners of investment real estate are noted for their reluctance to disclose net income, and a mailed request for same would almost certainly have resulted in the return of very few questionnaires. In addition, the experience of brokers and appraisers indicates a notable lack of consistency in the reporting of net income earned by real property. Operating and other expenses are recognized and recorded or estimated in almost as many different ways as there are numbers of owners. Lastly, while knowledge of net income would have been of considerable interest, it was not an essential part of this study. For these reasons, no attempt was made to obtain net income data.

Other information would also have been of considerable interest. For example, knowledge of mortgage financing might have helped to explain some of the variation in selling price. It is generally accepted among realtors and investors that an investment property which is favourably financed usually sells for a higher-than-usual price. As well, knowledge of land value would have been of interest. The knowledge of such things might have prompted some different treatment of the data in Chapter IV (i.e. different stratification). However, since such information could probably have been obtained only for a small percentage of the apartment properties and would therefore have been of little value, and because it was not considered essential to the reasonable completion of this study, no attempt was made to obtain same.

Interpretation of the data posed certain problems. The assurance fund value, which was assumed to be the selling price, may, in a few cases, not have been exactly the full price paid. In a small percentage of transfer registrations, the purchaser will swear that, in his opinion, the true market value of the property, at the date of sale, was something more or less than the actual price paid. In such cases, the sworn value is entered as the assurance fund value in lieu of the actual selling price. On the other hand, the assurance fund value may be the actual amount paid, but it may be an inflated figure reflecting a transaction in which the vendor took other property in trade.

In a few cases, basic construction was a mix of types. In other words, part of the apartment building was of masonry construction and the remainder (generally an addition) was of frame construction. In such instances, the writer categorized the building as being of masonry construction.

The age of each apartment building was taken to be the time which elapsed from the first year in which the property showed full assessment to the date of sale. In a few cases, due partly to time spent in construction and partly to assessment inspection practices, there was a lag of from one to two years between date of issuance of building permit and evidence of full assessment. In addition to this problem, there was the problem of classifying newly constructed buildings. With respect to these, all buildings which were found to be less than one year

old at the date of sale were assumed to be one year old.

The date of transfer registration was taken to be the date of sale. In the majority of cases, the sale would, in actuality, have taken place within the month or two prior to transfer registration. In the remaining few cases, the offer to purchase, and the acceptance, might have been signed several months (or, perhaps a year or more) prior to transfer registration.

A few of the returned questionnaires indicated a discrepancy in suite count. Part of this discrepancy is due to the fact that some apartment buildings contain one suite more than is permitted by City regulations. If existence of this "extra" suite has been discovered during an assessment inspection, the Assessor's Department records will indicate the true suite count. If it has not yet been discovered, the records will indicate the assumed number of suites only. The writer's records were altered whenever such a discrepancy was noted by a respondent.

In a few cases, the Assessor's Department records contained only the latest owner's name and address, thus preventing distribution of questionnaires to both vendor and purchaser.

Lastly, land and building assessment data, although recorded for each property, was found to be virtually meaningless for purposes of this study. The writer had intended to test the relationship between assessed value and selling price in addition to the relationship between gross rent and selling price. The former objective had to be abandoned because a large percentage of

the properties were new at the date of sale and consequently showed only partial assessments, or none, at that date. Full assessment often could not be found until one or two years after the sale occurred. In the case of many properties sold in 1970, the full assessment may not be shown on the tax roll until late 1971, or perhaps 1972.

In all, six hundred and ninety-two copies of the questionnaire were distributed by mail. Gross income data were obtained in respect of 157 sales of which 149 estimates were made by either the vendor or the purchaser[1], and 8 gross income estimates were received from other apparently knowledgeable parties. Thus, approximately 45 per cent (157/351) of the investigated apartment property sales became the final input data of this study.

For reasons of confidentiality, description of the raw data in the following pages does not include names, addresses, or legal descriptions.

Tables 1 through 7 illustrate basic grouping of the input data. The purpose of such grouping, or categorization, is to permit experimentation with samples which, in their makeup, reflect market attitudes and which, by their size, are adequate to yield statistically significant results.

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[1]In those few instances where both the vendor and purchaser replied, estimates of gross income were remarkably similar, and vendors' estimates were found to be higher than purchasers' estimates about as many times as they were lower. On the assumption that the vendor, being divorced from the property, could afford to be more objective, the writer chose to rely on the vendor's estimate rather than the purchaser's.

TABLE 1

DATA GROUPED BY TYPE OF CONSTRUCTION

Code	Construction	Frequency
(1)	Masonry	9
(2)	Frame	148
		157

Grouping by type of construction indicates that a large portion (nearly 95%) of the apartment buildings are of frame construction.

Grouping by age indicates that, at the date of sale, approximately 55% of the apartment buildings were one year old or newer, nearly 80% were less than 6 years old, and roughly 92% were less than 11 years old.

Grouping by number of suites indicates a predominance of the smaller and medium-sized walkup apartments. It must be noted that there was no market evidence of the transfer, resulting from a sale transaction, of any apartment properties of more than 100 suites in size. The 351 transfers, for which questionnaires were distributed, included 8 properties of from 49 suites to 100 suites in size. Questionnaires in respect of the latter were not returned.

Grouping by district is a reflection of the fact that, when charted on a map of the City, the apartment properties are clustered within ten major districts of the City. These districts are named, for purposes of this study, after the most central, or

TABLE 2

DATA GROUPED BY AGE

Code	Building Age	Frequency
(01)	New to 1 Year	86
(02)	2 Years	8
(03)	3 Years	11
(04)	4 Years	10
(05)	5 Years	9
(06)	6 Years	6
(07)	7 Years	5
(08)	8 Years	3
(09)	9 Years	2
(10)	10 Years	4
(11)	11 Years	2
(12)	12 Years	0
(13)	13 Years	1
(14)	14 Years	2
(15)	15 Years	2
(16)	16 Years	1
(17)	17 Years	1
(18)	18 Years	2
(19)	19 Years	2
(20)	20 Years	0
		157

TABLE 3

DATA GROUPED BY NUMBER OF SUITES

Code	Size of Apartment Building	Frequency
(04)	4 Suites	14
(06)	6 Suites	13
(08)	8 Suites	1
(09)	9 Suites	12
(10)	10 Suites	1
(11)	11 Suites	1
(12)	12 Suites	26
(14)	14 Suites	7
(15)	15 Suites	18
(16)	16 Suites	4
(17)	17 Suites	1
(18)	18 Suites	11
(19)	19 Suites	3
(20)	20 Suites	16
(21)	21 Suites	8
(22)	22 Suites	6
(23)	23 Suites	3
(24)	24 Suites	2
(26)	26 Suites	1
(27)	27 Suites	1
(30)	30 Suites	1
(33)	33 Suites	4
(36)	36 Suites	1

TABLE 3 (Continued)

Code	Size of Apartment Building	Frequency
(42)	42 Suites	1
(45)	45 Suites	1
		157

largest, subdivision within each district.[1]

Graphical representation of the dollars of income per room indicated two major central clusters with some notable tapering away at each end. This suggested the four broad categories shown.

Grouping by time is simply a division of the sample into sales which occurred before and after publication of proposals for tax reform, November 9th, 1969, by the Hon. E.J. Benson, Minister of Finance.

In grouping by date, the objective was that of securing groupings of adequate size, in consistent blocks of time, while allowing for elimination of those sales which occurred in the grey area in the weeks immediately preceding and following the announcement of the tax reform proposals.

Having examined, classified, and grouped the data in this fashion, it was possible to proceed with the research methodology

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[1]See map in Appendix, page 170.

TABLE 4

DATA GROUPED BY DISTRICT

Code	District	Frequency
(01)	Forest Heights	7
(02)	Delton	29
(03)	Jasper Place	23
(04)	Parkallen	15
(05)	Beverly	9
(06)	King Edward Park	17
(07)	Inglewood-Calder	10
(08)	Westmount-Glenora	4
(09)	Hudson Bay Reserve . . .	25
(10)	Central-Jasper Avenue . .	8
		157

which is outlined and discussed on the following pages.

METHODOLOGY

As has been stated, the central purpose of this study is to test the accuracy, and thus the reliability, of the gross rent multiplier as an appraisal device.

Firstly, the statistical relationship between selling price (the dependent variable) and gross rent (the independent variable) is gauged by means of regression and correlation analysis.

TABLE 5

DATA GROUPED BY INCOME PER ROOM

Code	Income per Room	Frequency
(1)	$312 and under	31
(2)	$313 to $362	65
(3)	$363 to $412	47
(4)	$413 and over	14
		157

TABLE 6

DATA GROUPED BY TIME

(Before or After Publication of Proposals For Tax Reform by the Hon. E.J. Benson, Minister of Finance, 1969)

Code	Time	Frequency
(1)	Before November 9, 1969 . . .	81
(2)	After November 9, 1969 . . .	76
		157

TABLE 7

DATA GROUPED BY DATE

Code	Sale Date	Frequency
(1)	Last Half 1968 . . .	41
(2)	First Half 1969 . .	28
(3)	Last Half 1969 . . .	32
(4)	First Half 1970 . .	19
(5)	Last Half 1970 . . .	37
		157

Regression equations based on a variety of data stratifications are calculated, coefficients are derived, and confidence interval widths at the 95 per cent level of confidence are determined.

Then, some of the resulting regression equations are applied to the gross income of selected properties in order to yield predicted values. These values are compared with the actual selling prices.

Lastly, in order to discover whether or not results will be significantly different through the use of some less-sophisticated, but similar, method of prediction, average multipliers and median multipliers are calculated in order to permit additional comparison of predicted and actual values.

The regression equations and related statistics are developed via a computer program. Input and output in respect of

the initial run appears on pages 168 and 169 of the Appendix. Additionally, in order to facilitate possible future replication or expansion of this study, pages 144 through 152 of the Appendix are devoted to brief discussion of the more pertinent statistical methods and formulae which underlie the applied program.

An additional objective of this study, as has also been noted earlier, is that of measuring the effect, if any, of the "proposals for tax reform" on the accuracy and applicability of the gross rent multiplier. Various tests are employed to determine whether or not the general level of apartment property values and the general level of multipliers differs significantly in the period after publication of Benson's white paper from the levels which were evident in the period prior to publication of the paper.

CHAPTER IV

RESULTS OF ANALYSIS

STRATIFICATION AND CONFIDENCE INTERVALS

In order to satisfactorily judge whether or not the gross rent multiplier is an accurate and reliable predictor some standard of accuracy must be selected.

Ratcliff, in an article which is based largely upon the findings of the Farish study, states his belief that there are no studies of actual appraiser performance nor standards of appraisal accuracy against which a given confidence interval might be judged.[1] He adds, however, the belief that

> ... most appraisers would be content to predict market value within 4% to 8%[2] of the actual selling price of the property in a group of properties with values which average nearly $300,000, as did our test sample.[3]

Since the Farish study employs the 95 per cent level of confidence, Ratcliff is saying, in effect, that most appraisers would be

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[1] Richard U. Ratcliff, "Don't Underrate the Gross Income Multiplier," Appraisal Institute Magazine, Vol. 14, Book 1 (1969-70), p. 19.

[2] This range, 4% to 8%, is reported by Ratcliff as being representative of the average per cent difference calculated by Farish in his comparison of predicted selling prices with actual selling prices.

[3] Ratcliff, "Don't Underrate the Gross Income Multiplier," p. 19.

satisfied to have some assurance that actual selling prices would be within 4% to 8% of predicted selling prices in at least 95 out of 100 appraisals. Another author offers the following observation:

> ... the range of error in the appraisal process normally will validate a latitude of 10% in the final conclusion of value.[4]

A form of support for this claim may be found in the attitude of local users of appraisal reports. Right-of-way buyers and others employed by the City of Edmonton and by the Government of Alberta often require two independent appraisals in respect of property rights which are to be expropriated or otherwise acquired. If the estimates of value are more than 10% apart, a third appraisal is generally demanded in the hope that the gap will be narrowed. In such instances, reliance is usually placed on the two estimates which show the least divergence of opinion. Occasionally, when the original two estimates are much more than 10% apart, neither appraisal is used. Representatives of major oil companies have similarly indicated their reluctance to consider estimates which are more than 10% apart, and at least one such company regards such a difference of opinion as an automatic requirement for two new independent appraisals.[5]

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[4]John J. Carney, "The Development and Use of Gross Income Multipliers," The Appraisal Journal, XXXI, No. 2 (1963), p. 225.

[5]These comments are based on personal experience gained by the writer of this study in his work as an independent fee appraiser in the Edmonton area.

Thus, although there is apparently no empirical evidence to support a claim that appraisers of real property would feel comfortable knowing that their estimates of selling price may be relied upon at least 95 times out of 100, the foregoing comments might be accepted as indication that such a level of assurance would be more than acceptable to most appraisers. However, this confidence level goes hand in hand with a confidence interval, and the appraiser's acceptance of a given regression equation will probably depend in large measure upon the relative width of the confidence interval.

As will be seen in the following pages, stratification fails to narrow the confidence interval to within ±10%. The confidence interval accompanying the regression equation which is provided by the entire sample of 157 properties, is ±25.62%. Successive stratification of the sample into smaller and more homogeneous groups permits narrowing of this interval to no better then ±10.73%.

The difference in construction (i.e. masonry vs. frame), as shown in the grouping of raw data in Chapter III, suggests the first data stratification. Differences in cost of construction, rent-producing capability, maintenance expenditure, lender attitudes, and provision for write-off of capital investment in improvements, are among the prime reasons for which these two types of buildings may be taken to constitute submarkets of the overall apartment property market.

Table 8 presents the result of stratification by type of construction.[6] This table shows the result of employing the regression equation provided by all 157 properties to obtain an estimate of value and an accompanying estimate of the width of the confidence interval. The regression equation, as contained in the Appendix, page 153, is seen to be

$$Yx = 5{,}323 + 6.560X.$$

The mean of X (i.e. the average of the 157 gross rents) is 23,761. Substitution of \$23,761 for X in the regression equation yields an estimated property value of \$161,195. Then, multiplication of the standard error of estimate, 21,073, by the standard normal deviate, 1.96[7], yields an estimated confidence interval width of $\pm$\$41,302. Since this latter figure could be misleading if shown strictly in terms of dollars, it is also shown as a per cent of estimated value. Thus, use of the regression equation provided by all 157 properties yields an estimate of value (where $X = \bar{X}$) of \$161,195 and an accompanying confidence interval of $\pm$\$41,302 or $\pm$25.62%. This says, in effect, that in 95 out of 100 appraisals which rely on the regression equation

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[6]Related regression and correlation statistics are contained in the Appendix. In addition to the regression equation, these include $\bar{X}$ (the sample mean gross rent), R^2 (the coefficient of determination), and the F-value (regression coefficient).

[7]For samples of 32 properties, or more, the standard errors of estimate are all multiplied by the same standard normal deviate (1.96 in the case of the 95% level of confidence). However, for smaller samples, and in those instances where the standard deviation of the population is unknown, the t-value, which varies with the number of degrees of freedom, must be used.

TABLE 8

CONFIDENCE INTERVALS
(Stratification by Type of Construction)

(1) Properties	(2) No.	(3) Yx, where $X=\bar{X}$ \$	(4) $\pm t_{N-2} S_{y \cdot x}$ \$	(5) (4) as % of (3)
All Properties	157	161,195	(1.96)(21,073) = 41,302	25.62
Masonry Only	9	187,180	(2.36)(20,532) = 48,455	25.89
Frame Only	148	159,611	(1.96)(21,106) = 41,368	25.92
Frame Only (excluding the three extremes)	145	158,916	(1.96)(17,441) = 34,185	21.51

$Yx = 5,323 + 6.560X$, the appraiser may be relatively confident that the actual selling prices will fall within 25.62% of his estimated values.[8] In other words, he may feel reasonably sure that he will be correct 95% of the time ±25.62%. There can be little doubt that such reliability would be considered unacceptable to most, or all, real property appraisers. Certainly this falls far beyond the ±10% latitude discussed earlier.

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[8]The confidence interval is not a constant band parallel to the regression line--it is at its minimum where $X = \bar{X}$ and curves outward as X moves away from the mean. Thus, estimates of confidence interval widths based on $\bar{X}$ are slightly misleading except in the case of very large samples. However, such estimates are sufficient for the purpose of this part of the study which is chiefly to discover the effect of stratification on the regression equation.

The fact that the regression coefficient (F) and the coefficient of determination (R^2) are relatively high is an indication of a fairly strong relationship between selling prices and gross rents and an indication that a large portion of variation in selling prices is explained by variation in gross rents. As may be seen on page 153 in the Appendix the found F-value greatly exceeds the table F-value and the R^2 is nearly equal to 1.00. If it were not for the unacceptably wide confidence interval, the regression equation might be considered useful in apartment property appraisal. In other words, the width of the confidence interval is a more important measure of the significance of the regression equation than is indication that the regression line shows goodness of fit. Thus, satisfactorily high R^2 and F-values are not, of themselves, sufficient evidence of reliability.

The remainder of Table 8 shows the change in width of the confidence interval which occurs when the data are stratified by type of construction. Of the 157 properties which make up the raw data of this analysis, only 9 contain buildings which are of masonry construction. These buildings vary considerably in size and age, and they are located in several parts of the City. A few of the 9 buildings, as was noted earlier, are partly of masonry construction and partly of frame construction. Because of the relative non-homogeneity of the masonry group and the smallness of sample size, it is not surprising that the width

of the confidence interval is not narrowed. As is indicated in Table 8, the confidence interval width is ±25.89%. The remaining properties, all of frame construction, make up over 90% of the total properties considered in this study, and they are almost certainly as varied in major characteristics (i.e. building size and age, location, etc.) as the overall sample. Again, it is not surprising to see no narrowing of the width of the confidence interval.

However, the confidence interval width is seen to narrow considerably when the properties which produce the three extreme multipliers[9] (all of frame construction) are excluded. Exclusion of obviously non-conforming data is compatible with normal appraisal practice--real property appraisers, in attempting to interpret the market, tend to discount, or disregard, market data which is grossly out of line with the majority of evidence. Perusal of the calculated gross rent multipliers indicates a general "spread" of multipliers running from a low of about 5 to a high of approximately 9. The multipliers produced by sales 049, 050, and 061 are considerably outside this general range and must be regarded as notably untypical of the market.

Further analysis deals strictly with those properties which contain buildings of frame construction[10] and, with the

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[9]See Appendix, page 139.

[10]Further study, via stratification, of the 9 properties with masonry structures, would obviously prove to be fruitless if for no other reason than that the sample is already of inadequate size for meaningful statistical inference.

exception of Table 9, analysis excludes the properties which produce the three extreme multipliers.

Table 9 shows the change in width of the confidence interval which occurs when the data are stratified by type of construction and by time. The sample of 148 frame properties is divided between those which sold prior to publication (November 9th, 1969) of Benson's proposals for tax reform and those which sold subsequent to publication of the proposals. The width of the confidence interval is increased from ±25.92% to ±26.79% when the sample is changed from the 148 properties to 78 properties which were sold prior to November 9th, 1969. The interval narrows to ±24.45% when the sample is changed from 148 properties to 70 properties which were sold after November 9th, 1969. These changes in width are so small that it would appear foolhardy to attempt to draw anything but the most general of conclusions therefrom. It might be inferred that publication of the proposals caused some of the late 1969 and 1970 purchasers to buy with just a shade more caution in reaction to the possibility of reduced future after-tax returns.[11] The tightening of the confidence interval suggests the value of an additional stratification--a form of stratification which will eliminate those sales which occurred in the "grey" area in the

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[11]The changes in width could be largely, or completely, the result of influence of the aforementioned three non-conforming sales. This probability is recognized in the analysis contained in Table 10.

TABLE 9

CONFIDENCE INTERVALS
(Stratification by Type of Construction
and by Time)

(1) Properties	(2) No.	(3) Y_x, where $X=\bar{X}$ $	(4) $\pm t_{N-2} S_{y \cdot x}$ $	(5) (4) as % of (3)
Frame, before publication of proposals for tax reform	78	138,553	(1.96)(18,934) = 37,110	26.79
Frame, after publication of proposals for tax reform	70	183,068	(1.96)(22,840) = 44,766	24.45

weeks immediately before and after newspaper announcement of the tax reform proposals. This is shown in Table 10.

Examination of the first portion of Table 10 indicates that elimination of the sales which occurred in the "grey" area results in an even greater increase in the width of the "before" interval and a considerably greater decrease in the width of the "after" interval. However, the top two sets of figures in Table 10 are the result of stratification without exclusion of the three non-conforming sales. The lower two sets of figures are the result of stratification which excludes the three non-conforming sales. What at first appears to be a sign of considerably altered relationship between gross income and price is found to have been caused by inclusion of two notably non-conforming sales in the "before" sample.[12]

[12]This somewhat dramatic difference in results spawned the earlier noted decision to cause succeeding tests to be based only on samples which do not include any of the three extreme transactions.

TABLE 10

CONFIDENCE INTERVALS
(Stratification by Type of Construction
and by Date)

(1) Properties	(2) No.	(3) Y_x, where $X=\bar{X}$ $	(4) $\pm t_{N-2} S_{y \cdot x}$ $	(5) (4) as % of (3)
Frame, last 1/2 of 1968 and first 1/2 of 1969	66	143,779	(1.96)(20,143) = 39,480	27.45
Frame, 1970	52	187,098	(1.96)(20,417) = 40,017	21.38
Frame, last 1/2 of 1968 and first 1/2 of 1969	64	142,398	(1.96)(14,176) = 27,785	19.51
Frame, 1970	52	187,098	(1.96)(20,417) = 40,017	21.38

Thus, since the lower figures are the significant results, it can be seen that the confidence interval width of ±21.51% (Table 8) is not meaningfully altered (or significantly narrowed or improved) by introducing "date" stratification.

Before proceeding to discuss the data shown in Table 11, it may be advisable to note that the more recent sales are of properties which are, on the average, more expensive. Thus, if the "after" sample had been found to produce a significantly narrower confidence interval, and if one can accept the premise that, in general, the expensive income-producing real property is purchased by professional investors while the less expensive

TABLE 11

CONFIDENCE INTERVALS
(Stratification by Type of Construction and by Age)

(1) Properties	(2) No.	(3) Y_x, where $X=\bar{X}$ \$	(4) $\pm t_{N-2} S_{y \cdot x}$ \$	(5) (4) as % of (3)
Frame, 10 years old or newer	135	165,067	(1.96)(18,012) = 35,304	21.39
Frame, 5 years old or newer	121	172,657	(1.96)(18,691) = 36,634	21.22
Frame, 1 year old or newer	84	191,620	(1.96)(20,655) = 40,484	21.13

income realty is purchased by less business-like investors (e.g. first-time investors such as the retired farmer), then such "before" and "after" results might reasonably have been regarded as a reflection of the more careful buying habits of the professional investor rather than some effect resulting from the tax reform proposals.

Table 11 shows the effect of stratification by type of construction and by age. Once again, the confidence interval width shown in Table 8 ($\pm$21.51%) is not meaningfully altered by introduction of a second level of stratification.

Table 12 shows the effect of stratification by type of construction and by number of suites. The groupings shown were selected largely because the data is distributed in a pattern

TABLE 12

CONFIDENCE INTERVALS
(Stratification by Type of Construction
and by Number of Suites)

(1) Properties	(2) No.	(3) Yx, where $X=\bar{X}$ $	(4) $\pm t_{N-2}S_{y \cdot x}$ $	(5) (4) as % of (3)
Frame, 4 to 6 suites	26	52,429	(2.064)(4,875) = 10,062	19.19
Frame, 4 to 11 suites	41	67,182	(1.96)(9,096) = 17,829	26.54
Frame, 12 suites	19	117,704	(2.110)(8,834) = 18,640	15.84
Frame, 12 to 16 suites	46	136,672	(1.96)(9,374) = 18,373	13.44
Frame, 18 to 20 suites	30	211,527	(2.048)(15,958) = 32,682	15.45
Frame, 18 to 24 suites	48	217,537	(1.96)(18,691) = 36,634	16.84
Frame, less than 18 suites	88	104,560	(1.96)(9,127) = 17,888	17.11
Frame, more than 17 suites	57	242,931	(1.96)(25,130) = 49,255	20.27

similar to the groupings, but also because these groupings may be taken to represent somewhat homogeneous submarkets. In other words, they not only provide samples of reasonably good size, but they separate the properties into "size" groups which appeal generally to buyers who are considered by brokers and lenders to exhibit somewhat different investor-behaviour characteristics. For example, it is generally believed that smaller apartment properties are purchased by husband and wife teams or other individuals who usually manage their own properties, and who often live in one of the suites and/or handle their own caretaker chores, while the larger apartment properties are purchased by wealthier individuals or investor syndicates and corporate entities, with management and caretaker responsibilities placed in the hands of professional personnel and/or salaried employees. Some of the resulting confidence intervals show considerable improvement; some show a considerable widening. Some small samples occur and the effect of the t-value is dramatically evident. In addition, in Tables 8 through 11 both the F and R^2 values were found to be relatively high; in the Table 12 tests, however, some of the R^2 values are found to be at a significantly low level.

The first of the groupings in Table 12, properties of lowest value, yields some improvement. The confidence interval is narrowed slightly (i.e. from $\pm 21.51\%$ to $\pm 19.19\%$). This improvement, however, is gained at the expense of some loss of

confidence in the strength of correlation--i.e. R^2 is found to have declined from 0.9614 to 0.6754, which means essentially that a user of the resulting regression equation must accept the fact that it credits only approximately 68% of the variation in price to variation in gross income.

Notable improvement in width of the confidence interval is shown in results of the third grouping. The confidence interval narrows from $\pm 21.51\%$ to $\pm 15.84\%$. Once again, however, the improvement is accompanied by a notable decline in the R^2 value--to 0.7714.

Of the eight groupings tested in Table 12, seven show a narrowing of the confidence interval. Four of these might be said to show an unreasonably low R^2 value. The lowest R^2 value of these four is 0.5302, which is the coefficient of determination attached to the sixth grouping (Frame, 18 to 24 suites). It may thus be seen that a relatively marked improvement in confidence interval width, of itself, can be a misleading indication of the usefulness of stratification by size. In fact, in this sixth grouping not only is the confidence interval markedly narrowed[13], but the F-value is significantly high. If reference were not made to the R^2 value, the regression equation might be accepted with more confidence than is warranted. Again, it must be noted that the appraiser requires assurance that the regression equation

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[13]It should be noted that there is no t-value effect on results produced by this grouping.

explains a satisfactory degree of variation as well as assurance of adequate predictive accuracy.

Before proceeding to discuss the data shown in Table 13, it may be useful to note the following observations. Firstly, although it is always dangerous to draw simple conclusions from statistical inference of this sort, it might nonetheless be fair to suspect that in those cases where the confidence interval is significantly narrowed, but the proportion of the variation which is accounted for by the regression equation is considerably lessened, two important and related factors may be at work. If the properties under consideration are among the more expensive of the original sample, it might be suspected that the more sophisticated investor is involved. This might account for the tighter purchase pattern. Additionally, with reference to Dorau's comments about the many benefits of ownership other than annual income[14], it may be suspected that the professional investor, more than other investors, will be aware of, and interested in, such other benefits. Secondly, with respect to the observation that some of the groupings results in small samples which requires use of values from a "t" distribution[15], it is important to recall that as the sample diminishes in size, other things being equal, the reliability of statistical inference

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[14]See Chapter II, page 35.

[15]The symbol t_{N-2} at the top of column (4) in the Tables represents the t-value for N-2 degrees of freedom. For samples of size 32, or larger, the value t_{N-2} is approximated by the standard normal deviate, which at the 95% level of confidence is 1.96.

diminishes. The t-values become rapidly larger as the sample size drops significantly below 32. The general effect of this is a greatly increased width in the confidence interval as the sample size declines. As analysis progresses, it will be seen that the sample which consists of less than 32 properties may not be as great an enemy of regression analysis (as an appraisal tool) as might be suspected.

Table 13 presents the results of stratification by type of construction and by district.[16] When charted on a map of the City, the apartment properties were seen to cluster within ten major districts. Division into these districts may be considered worthwhile for several reasons. Firstly, some of the districts have been built up as new subdivisions while others are older residential areas which are undergoing redevelopment. Secondly, some of the districts contain buildings which boast a fairly consistent level of quality (i.e. either generally high or generally low by comparison with the buildings of other districts). Lastly, most of the districts have their own distinctive advantage or disadvantage, generally speaking, in respect of nearness and access to the downtown area, shopping facilities, and sources of employment. To the extent that these are in fact features of some, or all,

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[16]When the data were initially gathered, it was noted that the 157 properties represented in excess of 50 subdivisions. With few exceptions, samples "by subdivision" were extremely small. Grouping by districts was required in order to permit meaningful statistical inference. See Appendix, page 170.

TABLE 13

CONFIDENCE INTERVALS
(Stratification by Type of Construction
and by District)

(1) Properties	(2) No.	(3) Yx, where $X=\bar{X}$ $	(4) $\pm t_{N-2} S_{y \cdot x}$ $	(5) (4) as % of (3)
Frame, Delton	28	134,365	(2.056)(13,151) = 27,038	20.12
Frame, Jasper Place	21	153,015	(2.093)(18,905) = 39,568	25.86
Frame, King Edward Park	16	134,703	(2.145)(14,105) = 30,255	22.46
Frame, Hudson Bay Reserve	23	201,970	(2.080)(19,627) = 40,824	20.21

of the districts, apartments within the districts may be considered to constitute submarkets of the overall apartment property market. The groupings shown in Table 13 were selected because they make up the four best samples in terms of size. The width of the confidence interval is widened in two of the groupings and is narrowed in the other two. The best result is obtained from those groupings which contain the most properties. In all four instances, the R^2 and F-values show fairly strong correlation of gross rents and selling prices. It should be noted once again that the results reflect the influence of t-values which

were required in the calculation of the width of the confidence intervals. The fact remains, however, that the confidence interval widths are not significantly altered and it must be concluded that while the results of Table 12 might indicate some value in stratification by size the results of Table 13 do not indicate material value in stratification by district.

Table 14 shows the effect of stratification by type of construction and by income per room. Income per room provides a rough, but readily available and easily calculated, measure of apartment quality. As was noted earlier, graphical representation of dollars of income per room indicated two major central clusters with some notable tapering away at each end. This suggested the four broad categories shown in Table 5, one of which provides a notably small sample. Of the four broad groupings, shown in the top portion of Table 14, three yield some narrowing of the confidence interval width, and it can be seen that the width becomes narrower as the income per room increases. If it can be assumed that the apartments which earn greater income per room are in fact higher quality structures, and if it can be assumed that the higher quality structures are generally the larger and more expensive buildings, this pattern of narrowing might be taken as support of the earlier noted contention that the more expensive income-producing properties are purchased, in the main, by professional or more-sophisticated purchasers.

TABLE 14

CONFIDENCE INTERVALS
(Stratification by Type of Construction and
by Income per Room)

(1) Properties	(2) No.	(3) Y_x, where $X=\bar{X}$ \$	(4) $\pm t_{N-2} S_{y \cdot x}$ \$	(5) (4) as % of (3)
Frame, \$312 and less	27	107,733	(2.060)(13,793) = 28,414	26.37
Frame, \$313 to \$362	62	147,203	(1.96)(15,379) = 30,143	20.48
Frame, \$363 to \$412	45	202,150	(1.96)(17,771) = 34,831	17.23
Frame, \$413 and over	11	173,681	(2.262)(11,792) = 26,673	15.36
Frame, \$313 to \$412	107	170,311	(1.96)(16,402) = 32,147	18.88
Frame, \$363 and over	56	196,570	(1.96)(19,754) = 38,718	19.70

The lower portion of Table 14 shows the results of combining the "middle" two categories and the "highest" two categories. Sample sizes are increased considerably while some improvement in width of the confidence interval is retained.

In all six instances, the R^2 and F-values show fairly strong correlation of gross rents and selling prices, and the results of the six tests indicate some value in stratification by income per room. Again, however, although the improvement brought about by such stratification is quite consistent it cannot be regarded as dramatic.

Although results obtained through stratification at two levels are generally disappointing--i.e. not highly significant--it is desirable to proceed with stratification at three, and more, levels in order to determine whether such further stratification will result in any notable additional narrowing of the confidence interval. While some of the stratification at two levels has narrowed the width of the confidence interval, the intervals are still relatively wide and the obtained regression equations are probably not practically useful. Further testing is also desired in order to permit comparison with the results of other studies.

Table 15 shows the result of three-level stratification. The first four tests are of samples which arise from stratification by construction, age, and size. In Table 8, the 145 frame properties provided a confidence interval of $\pm$21.51%.

TABLE 15

CONFIDENCE INTERVALS
(Three-level Stratification)

(1) Properties	(2) No.	(3) Yx, where $X=\bar{X}$ $	(4) $\pm t_{N-2} S_{y \cdot x}$ $	(5) (4) as % of (3)
Frame, 5 years old or newer, 4 to 6 suites	17	55,295	(2.131)(4,799) = 10,226	18.50
Frame, 5 years old or newer, 18 to 20 suites	28	213,027	(2.056)(16,540) = 34,006	15.96
Frame, 5 years old or newer, 18 to 24 suites	46	218,706	(1.96)(19,077) = 37,390	17.10
Frame, 5 years old or newer, more than 17 suites	55	244,739	(1.96)(25,597) = 50,171	20.50
Frame, 5 years old or newer, Delton	28	100,489	(2.056)(13,151) = 27,038	26.91
Frame, 5 years old or newer, Jasper Place	17	175,668	(2.131)(19,531) = 41,621	23.70
Frame, 5 years old or newer, King Edward Park	15	140,354	(2.160)(14,548) = 31,424	22.39

TABLE 15 (Continued)

(1) Properties	(2) No.	(3) Y_x, where $X=\bar{X}$ $	(4) $\pm t_{N-2} S_{y \cdot x}$ $	(5) (4) as % of (3)
Frame, 5 years old or newer, Hudson Bay Reserve	18	222,017	(2.120)(21,045) = 44,615	20.10
Frame, 5 years old or newer, $363 to $412	43	208,077	(1.96)(18,043) = 35,363	16.99
Frame, 5 years old or newer, $313 to $412	96	177,361	(1.96)(17,055) = 33,428	18.85
Frame, 5 years old or newer, $363 and over	52	206,261	(1.96)(20,045) = 39,287	19.05

In Table 11, the 121 frame apartments which are newer than six years provided a confidence interval of ±21.22%. In Table 12, the 26 frame apartments which are 4 to 6 suites in size provided a confidence interval of ±19.19%. Combination of all three levels in Table 15 provides a confidence interval of ±18.50%. However, as was discussed earlier, width of the confidence interval does not of itself tell the story. A narrower confidence interval will generally only be considered significant when the magnitude of the narrowing is notable and when the accompanying values of R^2 and F are satisfactorily high. Thus,

a more complete story is provided by the following tabulation.

Sample Size	Confidence Interval Width	R^2	Found F-value	Table F-value
145	±21.51%	0.9614	3,565.47	Less than 3.91
121	±21.22%	0.9560	2,587.13	Less than 3.92
26	±19.19%	0.6750	49.94	4.26
17	±18.50%	0.6913	33.59	4.54

The found F-value greatly exceeds the table F-value in each case, and each of the regression equations may be accepted as being significant (i.e. not likely the result of mere chance). However, the magnitude of the narrowing of the confidence interval represents only minor improvement. In addition, a notable decline in the value of R^2 accompanies both the three-level test and one of the two-level tests. Users of the regression equation provided by the three-level test would have to be aware of the fact that the regression equation accounts for (or explains) only approximately 69% of the variation in selling prices.

The second four tests in Table 15 are of samples which arise from stratification by construction, age, and district. The last three tests in Table 15 are of samples which arise from stratification by construction, age, and income per room. Of the entire eleven tests in Table 15, all show a significantly high F-value and only the first three show a questionable R^2 value.

However, when the confidence interval widths are compared with those provided by the two-level tests in Tables 12, 13, and 14, improvement is shown in three of the tests, virtually no change is shown in six of the tests, and a widening of the confidence interval results from the remaining two tests. It is interesting to note (as is indicated in the tabulated data which follows) that the good, indifferent, and bad "results" do not appear to be a function of sample size.

Table 15 Sample Size	Table 15 Confidence Interval	Tables 12, 13, and 14	Result (three-level test compared with earlier two-level test)
17*	±18.50%	±19.19%	narrowed
28*	15.96	15.45	same
46*	17.10	16.84	widened
55	20.50	20.27	same
28	26.91	20.12	widened
17	23.70	25.86	narrowed
15	22.39	22.46	same
18	20.10	20.21	same
43	16.99	17.23	narrowed
96	18.85	18.88	same
52	19.05	19.05	same

The first three tests (see asterisks) are those in which the R^2 value is notably low. The conclusion which appears to arise from this is that three-level stratification does not improve significantly on two-level stratification. Certainly the confidence interval is not narrowed to anything near ±10.00%.

Table 16 shows the results of a preliminary test of the effect of varying sample size. Three of the test groups shown

TABLE 16

CONFIDENCE INTERVALS
(Three-level Stratification--50% Random Sample)

(1) Properties	(2) No.	(3) Yx, where $X=\bar{X}$ $	(4) $\pm t_{N-2} S_{y \cdot x}$ $	(5) (4) as % of (3)
Frame, 5 years old or newer, 18 to 24 suites	17	214,886	(2.131)(15,616) = 33,277	15.49
Frame, 5 years old or newer, Hudson Bay Reserve	8	245,906	(2.447)(21,975) = 53,774	21.87
Frame, 5 years old or newer, $363 and over	23	181,835	(2.080)(19,333) = 40,214	22.12

in Table 15 were selected at random and the computer was directed to calculate a second set of regression and correlation statistics based on one-half the number of properties within each group (i.e. a random sampling of 50% of the properties).[17] The confidence interval is narrowed in the first of the three tests and is widened in the other two. An acceptably high F-value is produced by all three tests. The first test, however, produces

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[17]In several tests which called for the computer to random sample a certain percentage of properties within a group the percentage selected by the computer was notable and inexplicably different from the specified percentage.

a notably low R^2 value (0.6260). Thus, the sole improvement in confidence interval width occurs at the expense of confidence in correlation.

Table 17 shows the results of an additional test of the effect of varying sample size. This test centers on two three-level groupings--one selected from Table 15 and the other chosen at random from three-level groupings for which regression and correlation statistics had not yet been acquired. The computer was directed to calculate regression and correlation statistics for each test group (with randomly selected samples) so that confidence interval widths could be calculated at 10% intervals. With respect to the first grouping, the width of confidence interval does not vary significantly for sample sizes 56 to 96. It is interesting to note, however, that the smaller samples, 18 to 42, produce a considerably narrower confidence interval despite the fact that in two instances smallness of sample size necessitates use of the t-value. It is also interesting to note that the confidence interval width produced by the smallest sample, 9, which is affected most by introduction of the t-value, is only slightly wider than those produced by the 5 largest samples. With respect to the second grouping, the width of the confidence interval is found to narrow significantly and quite consistently as the sample size declines (i.e. from ±18.02% for sample size 47 to ±11.82% for

TABLE 17

CONFIDENCE INTERVALS
(Three-level Stratification--Comparison of Varying, Randomly Selected, Sample Sizes)

(1) Properties	(2) No.	(3) Yx, where $X=\bar{X}$ $	(4) $\pm t_{N-2} S_{y \cdot x}$ $	(5) (4) as % of (3)
Frame, 5 years old or newer, $313 to $412	9(10%)	141,862	(2.365)(12,352) = 29,212	20.59
	18(20%)	151,346	(2.120)(11,299) = 23,954	15.83
	29(30%)	158,834	(2.051)(12,075) = 24,778	15.60
	35(40%)	156,078	(1.96)(12,077) = 23,671	15.17
	42(50%)	155,890	(1.96)(14,209) = 27,850	17.86
	56(60%)	162,729	(1.96)(16,080) = 31,517	19.37
	67(70%)	164,204	(1.96)(16,670) = 32,673	19.90
	78(80%)	169,156	(1.96)(16,685) = 32,703	19.33
	82(90%)	169,483	(1.96)(16,735) = 32,800	19.35
	96(100%)	177,361	(1.96)(17,055) = 33,428	18.85

TABLE 17 (Continued)

(1) Properties	(2) No.	(3) Yx, where $X=\bar{X}$ $	(4) $\pm t_{N-2} S_{y \cdot x}$ $	(5) (4) as % of (3)
Frame, more than 17 suites, $313 to $412	4(10%)	214,835	(4.303)(5,902) = 25,396	11.82
	8(20%)	220,409	(2.447)(9,665) = 23,649	10.73
	14(30%)	217,911	(2.179)(13,496) = 29,409	13.50
	15(40%)	218,716	(2.160)(13,902) = 30,030	13.73
	18(50%)	219,725	(2.120)(15,893) = 33,693	15.33
	23(60%)	230,073	(2.080)(17,228) = 35,835	15.58
	29(70%)	225,370	(2.052)(19,265) = 39,532	17.54
	37(80%)	226,094	(1.96)(20,045) = 39,289	17.38
	39(90%)	227,029	(1.96)(19,937) = 39,076	17.21
	47(100%)	238,067	(1.96)(21,894) = 42,913	18.02

sample size 4.[18] The broad conclusion which would seem to arise from this is that sample sizes of less than 32 properties, in this study, may not be a notable enemy of regression analysis (as may at first be suspected), and any observed lack of reliability in use of the regression equation might better be considered an inherent defect in the data, or impropriety of the assumption that the relationship between gross incomes and selling prices is significant in real property appraisal, rather than the result of inadequate sample sizes.

Tables 18 and 19 show the results of stratification at four and five levels. In many cases sample sizes become unreasonably small (non-existent in one instance) with the result that confidence intervals widen considerably and/or the R^2 and F-values indicate poor correlation between gross incomes and selling prices. Probably only 4 of the 20 groupings can be said to produce reasonably good results--i.e. only in 4 of the groupings does the resulting confidence interval[19] remain in the

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[18]In the first grouping the R^2 and F-values may be considered acceptably high for all sample sizes; in the second grouping the R^2 and F-values may be considered acceptably high for all but the two smallest samples (i.e. sample sizes 4 and 8). This is, incidentally, the first point in examination of the effect of stratification at which the found F-value fails to equal or exceed the table F-value.

[19]The width of the confidence interval in a given sample is the result of three influencing factors--the standard error of estimate, the sample size, and the level of confidence. With respect to the first of these, the choice of closely comparable properties via intelligent stratification will act to minimize the inherent variation in the grouping. With respect to the

TABLE 18

CONFIDENCE INTERVALS
(Four-level Stratification)

(1) Properties	(2) No.	(3) Yx, where $X=\bar{X}$ $	(4) $\pm t_{N-2} S_{y \cdot x}$ $	(5) (4) as % of (3)
Frame, 5 years old or newer, 4 to 6 suites, Delton	6	55,557	(2.776)(3,777) = 10,485	18.87
Frame, 5 years old or newer, 4 to 6 suites, King Edward Park	4	60,563	(4.303)(4,857) = 20,884	34.48
Frame, 5 years old or newer, 4 to 6 suites, Hudson Bay Reserve	0	----	----	----
Frame, 5 years old or newer, 18 to 24 suites, Delton	5	195,200	(3.182)(23,530) = 74,872	38.36
Frame, 5 years old or newer, 18 to 24 suites, King Edward Park	6	219,410	(2.776)(14,795) = 41,068	18.71
Frame, 5 years old or newer, 18 to 24 suites, Hudson Bay Reserve	11	218,826	(2.262)(19,729) = 44,628	20.39

TABLE 18 (Continued)

(1) Properties	(2) No.	(3) Yx, where $X=\bar{X}$ $	(4) $\pm t_{N-2} S_{y \cdot x}$ $	(5) (4) as % of (3)
Frame, 10 years old or newer, 18 to 24 suites, Delton	5	195,200	(3.182)(23,530) = 74,872	38.36
Frame, 10 years old or newer, 18 to 24 suites, Hudson Bay Reserve	12	215,691	(2.228)(18,883) = 42,071	19.51
Frame, 10 years old or newer, 18 to 24 suites, $313 to $412	41	218,455	(1.96)(19,611) = 38,437	17.59
Frame, 10 years old or newer, 18 to 24 suites, $363 and over	22	227,715	(2.086)(15,823) = 33,006	14.50
Frame, 10 years old or newer, 18 to 24 suites, Before proposals for tax reform	19	206,524	(2.110)(20,538) = 43,335	20.98
Frame, 10 years old or newer, 18 to 24 suites, After proposals for tax reform	22	224,117	(2.086)(19,302) = 40,264	17.97

TABLE 18 (Continued)

(1) Properties	(2) No.	(3) Yx, where $X=\bar{X}$ $	(4) $\pm t_{N-2}S_{y \cdot x}$ $	(5) (4) as % of (3)
Frame, More than 17 suites, $363 and over, Before proposals for tax reform	9	243,012	(2.365)(17,077) = 40,387	16.61
Frame, More than 17 suites, $363 and over, After proposals for tax reform	16	272,803	(2.145)(24,872) = 53,348	19.55

previous pattern (or show some improvement) coincident with a significantly high R^2 and F-value. Thus it would appear that stratification, in this study, is impractical beyond three levels. It would also, as has been indicated, seem to be a fact that even in those cases where stratification shows consistency in

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second, efforts to secure samples of 32, or more, properties will minimize reliance on the higher t-values. With respect to the third, the appraiser can choose to employ a level of confidence less than 95%, say 66.26%. Dealing in this way with the problems of sample size presents certain problems, however. The data may often be limited in number with the result that even moderate stratification will yield a small sample. Also, the choice of a lower level of confidence is rarely a solution. Selection of the 66.26% level of confidence virtually halves the standard normal deviate (1.00 as against 1.96, for the large samples) and this alone will act to narrow the width of the confidence interval by approximately 50%. However, in exchange for this gain, the appraiser will have confidence that the actual value will fall within the calculated interval only in approximately 2 out of every 3 appraisals as against the previous 95% level of confidence.

TABLE 19

CONFIDENCE INTERVALS
(Five-level Stratification)

(1) Properties	(2) No.	(3) Y_x, where $X=\bar{X}$ $	(4) $\pm t_{N-2} S_{y \cdot x}$ $	(5) (4) as % of (3)
Frame, 10 years old or newer, More than 17 suites, Hudson Bay Reserve, Before proposals for tax reform	5	238,024	(3.182)(19,621) = 62,434	26.23
Frame, 10 years old or newer, More than 17 suites, Hudson Bay Reserve, After proposals for tax reform	8	217,162	(2.447)(19,807) = 48,469	22.32
Frame, More than 17 suites, $363 and over, Westmount-Glenora and Hudson Bay Reserve, Before proposals for tax reform	4	268,542	(4.303)(10,519) = 45,263	16.85
Frame, More than 17 suites, $363 and over, Westmount-Glenora and Hudson Bay Reserve, After proposals for tax reform	10	276,437	(2.306)(28,451) = 65,608	23.73

TABLE 19 (Continued)

(1) Properties	(2) No.	(3) Yx, where $X=\bar{X}$ \$	(4) $\pm t_{N-2}S_{y \cdot x}$ \$	(5) (4) as % of (3)
Frame, More than 17 suites, \$363 and over, Parkallen and King Edward Park, Before proposals for tax reform	3	221,688	(12.706)(24,314) = 308,934	139.35
Frame, More than 17 suites, \$363 and over, Parkallen and King Edward Park, After proposals for tax reform	2	----	----	----

narrowing the confidence interval the magnitude of the narrowing falls so much outside of the apparently acceptable ±10.00% limit that the stratification is of no value.

PREDICTIONS BASED ON REGRESSION ANALYSIS

Although considerable difficulty has been encountered in narrowing the confidence intervals of the predictive equations through sample stratification, the question of the value of stratification and the question of the reliability of the equations can be answered more fully. In this regard, the

following pages are addressed to the question: If an appraiser had used some of the calculated regression equations, how close would he have come in his predictions to the prices actually received?

Table 20 shows the result of the first comparison of predicted values with actual selling prices. The test involves the 46 properties contained in the third grouping in Table 15. Ten properties were selected at random from among the 46. Then, using the regression equation which is produced by the 46, the gross incomes of the 10 randomly selected properties are made to yield 10 predicted values. These 10 predicted values are then compared with the actual selling prices in order to determine the "difference" in dollars and as a percentage of the actual price.[20] Lastly, the average of the percentage differences is calculated in order to estimate the average accuracy of the predicting equation.[21] As is indicated in the Table, the average % difference is 8.83%.

In an extension of the test, the regression equation produced by a 50% random sampling of the same grouping (i.e. Frame, 5 years old or newer, 18 to 24 suites) is used to predict the

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[20]Although the differences are sometimes positive and sometimes negative, signs are ignored; the point of interest is the magnitude of the difference between the predicted and actual values, not whether the difference is high or low.

[21]This may also be explained by stating that the "% differences" are averaged so that an average of the variation may be shown. The average % difference is thus an average of percentages, not the average absolute difference expressed as a percentage of the mean value. Such averaging of percentages avoids the weighting effect that would obviously result from any notable mix of property sizes in a sample.

TABLE 20

PREDICTED VERSUS ACTUAL VALUES
(Stratification by Type of Construction, Age, and Number of Suites)

Parent Sample: 46 (Frame, 5 years old or newer, 18 to 24 suites--see Table 15)
Regression Equation: $Yx = 42,634 + 5.399X$
$Sy \cdot x = 19,077$
$R^2 = 0.5073$

Predicted	Actual	Difference	% Difference
188,407	162,000	26,407	16.30
222,421	235,000	12,579	5.35
249,956	271,200	21,244	7.83
204,604	228,500	23,896	10.46
226,632	209,000	17,632	8.44
198,125	223,000	24,875	11.15
178,825	198,000	19,175	9.68
217,562	195,000	22,562	11.57
230,519	241,000	10,481	4.35
269,392	260,000	9,392	3.61
	Average % Difference		8.83

50% Sample: 17 (See Table 16)
Regression Equation: $Yx = -2,545 + 6.768X$
$Sy \cdot x = 15,616$
$R^2 = 0.6260$

Predicted	Actual	Difference	% Difference
180,191	162,000	18,191	11.23
222,829	235,000	12,171	5.18
257,346	271,200	13,854	5.11
200,495	228,500	28,005	12.26
228,108	209,000	19,108	9.14
192,373	223,000	30,627	13.73
166,926	198,000	31,074	15.69
216,738	195,000	21,738	11.15
232,981	241,000	8,019	3.33
281,711	260,000	21,711	8.35
	Average % Difference		10.52

selling price of the same 10 properties. Once again, the predicted values are compared with the actual selling prices in order to estimate the average accuracy of the predicting equation. In this test, as is indicated in the Table, the average % difference is 10.52%.

The result of the first test says, in effect, that use of the regression equation $Yx = 42{,}634 + 5.399X$[22] to predict the value of the 10 randomly selected properties[23] is accurate, on the average, to within 8.83%. If this pattern could be expected to persist, the appraiser who relies upon this regression equation to predict the selling prices of other similar properties could assume that his estimates of value will be correct, on the average, $\pm$8.83%. This is, of course, within the apparently acceptable $\pm$10.00% limit. However, this conclusion may be considered misleading for at least two reasons. Firstly, the 8.83% interval is an average and, as such, is the result of some predictions which missed their mark by more than 8.83%. To be exact, five of the predicted values were more than 8.83% different from the actual prices, and four were more than 10.00% different from the actual prices. Thus, the regression equation may be said to be accurate to within 8.83% in 50% of the

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[22]This equation, produced by a grouping selected at random from Table 15, is accompanied by an R^2 value of 0.5073.

[23]This randomness was achieved through use of tables of random numbers.

predictions. It may also be said to provide value indications with a % difference greater than 10.00% in 40% of the appraisals. Secondly, data in respect of the 10 test properties were used in calculation of the regression equation, and the equation might therefore be suspected of bias. The conclusion which might arise from all of this is that the claim of accuracy, on the average, to within 8.83%, is not necessarily as significant as it may at first appear to be.

The result of the second test says, in effect, that use of the regression equation $Yx = -2{,}545 + 6.768X$ (produced by a much smaller sample)[24] to predict the value of the 10 randomly selected properties is accurate, on the average, to within 10.52%. While the result appears to suggest that less reliability accompanies the smaller sample, one such test must be regarded as highly inconclusive. Clearly, additional tests of predictive accuracy are required.

Table 21 shows the result of a further comparison of predicted values with actual selling prices. This test involves the 47 properties contained in the second grouping in Table 17. Again, 10 properties were selected at random from among the 47, and the regression equation produced by all 47 properties is used to predict the selling prices of the 10 properties.[25]

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[24]Where $X = \bar{X}$, this equation shows a confidence interval of $\pm 15.49\%$, but this is accompanied by an R^2 value of only 0.6260.

[25]This equation shows a confidence interval, where $X = \bar{X}$, of $\pm 18.02\%$. The found F-value exceeds the table F-value and $R^2 = 0.8732$.

TABLE 21

PREDICTED VERSUS ACTUAL VALUES
(Stratification by Type of Construction, Number of Suites, and Income per Room)

Parent Sample: 47 (Frame, more than 17 suites, \$313 to \$412--see Table 17)
Regression Equation: $Yx = 22{,}442 + 5.972X$
$Sy \cdot x = 21{,}894$
$R^2 = 0.8732$

Predicted	Actual	Difference	% Difference
190,136	161,000	29,136	18.10
251,767	271,200	19,433	7.17
225,251	222,000	3,251	1.46
194,436	223,000	28,564	12.81
219,518	225,000	5,482	2.44
215,935	195,000	20,935	10.74
235,642	185,700	49,942	26.89
214,358	248,800	34,442	13.84
258,933	247,500	11,433	4.62
402,261	353,100	49,161	13.92
	Average % Difference		10.30

50% Sample: 18 (See Table 17)
Regression Equation: $Yx = 41{,}791 + 5.366X$
$Sy \cdot x = 15{,}893$
$R^2 = 0.8314$

Predicted	Actual	Difference	% Difference
192,468	161,000	31,468	19.55
247,845	271,200	23,355	8.61
224,020	222,000	2,020	0.91
196,332	223,000	26,668	11.96
218,869	225,000	6,131	2.72
215,649	195,000	20,649	10.59
233,357	185,700	47,657	25.66
214,233	248,800	34,567	13.89
254,285	247,500	6,785	2.74
383,069	353,100	29,969	8.49
	Average % Difference		10.51

Comparison of the predicted values with the actual selling prices indicates that the equation is accurate, on the average, to $\pm$10.30%.

Again, as an extension of the test, the regression equation[26] produced by a 50% random sampling of the same grouping (also Table 17) is used to predict the selling price of the 10 properties. The average % difference is found to be 10.51%, not significantly different from the above noted 10.30%.

Table 22 shows the results obtained when the test is repeated using the other grouping in Table 17 (i.e. 96 properties). Again, the result obtained via application of the equation produced by the 50% sample is extremely close to that obtained via application of the equation produced by all 96 properties. However, unlike the results of Table 21, the average % difference is found to be notably low--4.89% and 4.87%. The superior results of this table are explained by the correlation statistics which show that the respective confidence intervals, where $X = \bar{X}$, are $\pm$18.85% and $\pm$17.86% while the values of R^2 equal 0.9520 and 0.9564.[27]

In addition to the low average % difference, the results of Table 22 reveal that only one of the 10 predicted values differs from the actual value by more than 10.00%. In the interest

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[26] This equation shows a confidence interval, where $X = \bar{X}$, of $\pm$15.33%. The found F-value exceeds the table F-value, and R^2 = 0.8314.

[27] Both found F-values exceed the table F-values.

TABLE 22

PREDICTED VERSUS ACTUAL VALUES
(Stratification by Type of Construction, Age, and Income per Room)

Parent Sample: 96 (Frame, 5 years old or newer, \$313 to \$412--see Table 17)
Regression Equation: $Yx = 8{,}435 + 6.284X$
$Sy{\cdot}x = 17{,}055$
$R^2 = 0.9520$

Predicted	Actual	Difference	% Difference
61,221	59,000	2,221	3.76
88,367	82,000	6,367	7.76
90,630	102,000	11,370	11.15
140,248	130,000	10,248	7.88
126,071	130,000	3,929	3.02
164,907	164,000	907	0.55
157,240	153,975	3,265	2.12
187,152	171,000	16,152	9.45
202,611	201,500	1,111	0.55
214,550	209,000	5,550	2.66
	Average % Difference		4.89

50% Sample: 42 (See Table 17)
Regression Equation: $Yx = 8{,}833 + 6.262X$
$Sy{\cdot}x = 14{,}209$
$R^2 = 0.9564$

Predicted	Actual	Difference	% Difference
61,434	59,000	2,434	4.13
88,486	82,000	6,486	7.91
90,740	102,000	11,260	11.04
140,185	130,000	10,185	7.83
126,058	130,000	3,942	3.03
164,757	164,000	757	0.46
157,117	153,975	3,142	2.04
186,924	171,000	15,924	9.31
202,329	201,500	829	0.41
214,227	209,000	5,227	2.50
	Average % Difference		4.87

of determining whether or not random selection of the 10 test properties resulted by chance in a grouping of properties significantly more comparable than those of the parent sample as a whole, the parent regression equation (i.e. $Yx = 8{,}435 + 6.284X$) was used to predict the selling price of all 96 properties. The results, shown on pages 163 thru 167 of the Appendix, indicate that this had in fact occurred.

Tables 23 and 24 show the results of two additional tests which are similar in nature but which differ notably in the method of sampling.

The first test in Table 23 involves the 43 properties contained in the ninth grouping in Table 15. As in the preceding tables, 10 properties were selected at random from among the 43 and the regression equation produced by all 43 was used to predict the value of each of the 10 properties. The average % difference is found to be 6.86%, and three of the 10 predicted values are found to differ from the actual selling prices by more than 6.86%. These three differ, in fact, by considerably more than 10.00%. Thus, the regression equation[28] may be said to be accurate to within ±6.86% in 70% of the predictions; in the remaining 30%, predicted values can be expected to differ from actual selling prices by considerably more than 10.00%.

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[28]This equation shows a confidence interval, where $X = \bar{X}$, of ±16.99%. The found F-value exceeds the table F-value, and $R^2 = 0.9595$.

TABLE 23

PREDICTED VERSUS ACTUAL VALUES
(Stratification by Type of Construction, Age, and Income per Room)

Parent Sample: 43 (Frame, 5 years old or newer, \$363 to \$412--see Table 15)
Regression Equation: $Yx = 12,201 + 6.241X$
$Sy \cdot x = 18,043$
$R^2 = 0.9595$

Predicted	Actual	Difference	% Difference
57,885	50,000	7,885	15.77
81,476	70,000	11,476	16.39
94,208	89,000	5,208	5.85
147,007	151,500	4,493	2.97
164,606	156,000	8,606	5.52
156,368	157,000	632	0.40
159,426	160,000	574	0.36
205,048	201,500	3,548	1.76
199,131	233,500	34,369	14.72
229,388	241,000	11,612	4.82
	Average % Difference		6.86

Random Sample: 15 (See Appendix, Page 162)
Regression Equation: $Yx = 10,604 + 6.397X$
$Sy \cdot x = 18,280$
$R^2 = 0.9594$

Predicted	Actual	Difference	% Difference
57,430	50,000	7,430	14.86
81,611	70,000	11,611	16.59
94,661	89,000	5,561	6.36
148,779	151,500	2,721	1.80
166,819	156,000	10,819	6.94
158,375	157,000	1,375	0.88
161,509	160,000	1,509	0.94
208,271	201,500	6,771	3.36
202,207	233,500	31,293	13.40
233,200	241,000	7,780	3.23
	Average % Difference		6.84

TABLE 24

PREDICTED VERSUS ACTUAL VALUES
(Stratification by Type of Construction, Age, and Number of Suites)

Parent Sample: 46 (Frame, 5 years old or newer, 18 to 24 suites--see Table 15)
Regression Equation: $Yx = 42{,}634 + 5.399X$
$Sy \cdot x = 19{,}077$
$R^2 = 0.5073$

Predicted	Actual	Difference	% Difference
196,182	171,000	25,182	14.73
185,492	187,450	1,958	1.04
225,336	237,000	11,664	4.92
204,604	228,500	23,896	10.46
225,984	222,000	3,984	1.79
208,686	207,000	1,686	0.81
204,604	205,000	396	0.19
177,825	198,000	20,175	10.19
226,632	230,000	3,368	1.46
247,256	265,000	17,744	6.70
	Average % Difference		5.23

Random Sample: 15 (See Appendix, page 162)
Regression Equation: $Yx = 24{,}154 + 5.787X$
$Sy \cdot x = 23{,}900$
$R^2 = 0.5173$

Predicted	Actual	Difference	% Difference
188,736	171,000	17,736	10.37
177,278	187,450	10,172	5.43
219,986	237,000	17,014	7.18
197,764	228,500	30,736	13.45
220,681	222,000	1,319	0.59
202,139	207,000	4,861	2.35
197,764	205,000	7,236	3.53
169,060	198,000	28,940	14.62
221,375	230,000	8,625	3.75
243,481	265,000	21,519	8.12
	Average % Difference		6.94

The second test in Table 23 shows the effect of predicting the 10 selling prices via a regression equation which is obtained from a smaller randomly selected sample which does not include any of the 10 test properties. The 10 test properties were first selected at random, as indicated above, from the parent sample of 43 properties. Then, 15 properties were selected (again, at random) from the remaining 33 properties. Regression analysis of the 15 properties provides the equation $Yx = 10{,}604 + 6.397X$.[29] When this equation is used to predict the selling prices of the 10 test properties, the average % difference between the predicted values and the actual values is seen to be 6.84%, remarkably close to the average % difference shown in the first test in Table 23. Again, however, it is observed that three of the predicted values differ from the actual by considerably more than 10.00%.

The first test in Table 24 involves the 46 properties contained in the third grouping in Table 15. As in the preceding tables, 10 properties were selected at random from among the 46 and the regression equation[30] produced by all 46 was used to predict the value of each of the 10 properties. The average % difference is found to be 5.23%, and three of the 10 predicted

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[29]Regression and correlation statistics in respect of the sample which produced this equation are presented in the Appendix, page 162.

[30]This equation shows a confidence interval, where $X = \bar{X}$, of $\pm$17.10%. The found F-value exceeds the table F-value, but R^2 is a relatively low 0.5073.

values are found to differ from the actual selling prices by considerably more than 10.00%.

The second test in Table 24 shows the effect, again, of predicting the 10 selling prices via a regression equation which is obtained from a randomly selected sample (15 properties) which does not include any of the 10 test properties.[31] The average % difference is seen to be 6.94%, slightly greater than the average % difference shown in the first test in Table 24. Again, it is observed that 3 of the predicted values differ from the actual by more than 10.00%.

THE AVERAGE MULTIPLIER

In this part of the study, predicted selling prices are again compared with actual selling prices, but the selling prices are predicted via the more easily calculated average multipliers. For example, in the first part of Table 25 the 52 properties of the parent sample are made to yield an average gross rent multiplier (i.e. arithmetic mean of the 52 multipliers). This average multiplier is then applied to the gross incomes of the 10 randomly selected test properties in order to produce 10 predicted selling prices. Then, as in the previous section, the predicted selling prices are compared with the actual selling prices and an average % difference is calculated.

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[31] Regression and correlation statistics in respect of the sample which produced this equation are presented in the Appendix, page 162.

TABLE 25

PREDICTED VERSUS ACTUAL VALUES
(Stratification by Type of Construction, Age, and Income per Room)

Parent Sample: 52 (Frame, 5 years old or newer, $363 and over--see Table 15)
Average Multiplier: 6.6955

Predicted	Actual	Difference	% Difference
56,242	57,000	758	1.33
99,629	102,000	2,371	2.32
166,718	164,000	2,718	1.66
170,253	168,000	2,253	1.34
226,576	237,000	10,424	4.40
235,816	229,300	6,516	2.84
191,223	200,000	8,777	4.39
215,167	248,800	33,633	13.52
361,959	339,225	22,734	6.70
472,823	471,000	1,823	0.39
	Average % Difference		3.89

Random Sample: 15
Average Multiplier: 6.7797

Predicted	Actual	Difference	% Difference
56,949	57,000	51	0.09
100,882	102,000	1,118	1.10
168,815	164,000	4,815	2.94
172,394	168,000	4,394	2.62
229,425	237,000	7,575	3.20
238,781	229,300	9,481	4.13
193,628	200,000	6,372	3.19
217,872	248,800	30,928	12.43
366,510	339,225	27,285	8.04
478,769	471,000	7,769	1.65
	Average % Difference		3.94

It is highly probable that few practicing appraisers possess a working knowledge of the techniques of statistical inference, and it is even more probable that few appraisers have access to computer facilities. It is therefore desirable to discover the difference, if any, in the predictive ability of the average multiplier as against that of the regression equation.

The first test in Table 25 involves the 52 properties contained in the eleventh grouping in Table 15. Ten properties were selected at random from among the 52 and the average of the 52 gross rent multipliers (6.6955) was applied to the gross income of the 10 properties in order to yield 10 estimates of value. These estimates (i.e. predicted values) are then compared with the actual selling prices. The average % difference is found to be only 3.89%. Six of the predicted values are seen to be exceptionally close to the actual values and only one differs from the actual by more than 10.00%.

The second test in Table 25 shows the effect of predicting the 10 selling prices via an average multiplier which is obtained from a smaller randomly selected sample which does not include any of the 10 test properties. The 10 test properties were first selected at random, as indicated above, from the parent sample of 52 properties. Then, 15 properties were selected (again, at random) from the remaining 42 properties, and the average of the 15 gross rent multipliers was calculated. Application of this average multiplier (6.7797) to the gross

incomes of the 10 test properties produced 10 predicted values, and comparison of the 10 predicted values with the 10 actual selling prices indicated an average % difference of 3.94%. This is very close to the result of the first test in Table 25, and it can be seen that once again only one of the predicted values differed from the actual by more than 10.00%.

Table 26 shows the results of a repeat of the tests contained in Table 25. The parent sample consists of the 41 properties contained in the ninth grouping of Table 18. The average % difference is 9.46% in the first test and 9.48% in the second test. In each of the tests, 5 of the predicted values differ from the actual values by more than 10.00%.[32]

Tables 27 and 28 show the results of tests which are identical to those of Tables 25 and 26 but which utilize the data of Tables 23 and 24. The purpose is to permit comparison of results obtained via the average multiplier with results obtained via the regression equation when the tests are centered

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[32] The considerable difference in predictive reliability of the average multiplier in Table 25 as against that of the average multiplier in Table 26 is explained by considerable difference in the strength of relationship between gross incomes and selling prices. Regression and correlation analysis of the data in Table 25 (first test) indicates a confidence interval, where $X = \bar{X}$, of $\pm 19.05\%$, and an R^2 value of 0.9639. Regression and correlation analysis of the data in Table 26 (first test) indicates a confidence interval, where $X = \bar{X}$, of $\pm 17.59\%$, and an R^2 value of 0.5314. The found F-value exceeds the table F-value in each case.

TABLE 26

PREDICTED VERSUS ACTUAL VALUES
(Stratification by Type of Construction, Age,
Number of Suites, and Income per Room)

Parent Sample: 41 (Frame, 10 years old or newer, 18 to 24 suites, $313 to $412--see Table 18)
Average Multiplier: 6.6580

Predicted	Actual	Difference	% Difference
191,750	170,000	21,750	12.79
189,354	171,000	18,354	10.73
180,165	181,000	835	0.46
199,740	228,500	28,760	12.59
199,740	205,000	5,260	2.57
237,691	186,000	51,691	27.79
216,518	229,100	12,582	5.49
213,961	248,800	34,839	14.00
269,489	274,100	4,611	1.68
263,657	247,500	16,157	6.53
	Average % Difference		9.46

Random Sample: 15
Average Multiplier: 6.6495

Predicted	Actual	Difference	% Difference
191,506	170,000	21,506	12.65
189,112	171,000	18,112	10.59
179,935	181,000	1,065	0.59
199,485	228,500	29,015	12.70
199,485	205,000	5,515	2.69
237,387	186,000	51,387	27.63
216,242	229,100	12,858	5.61
213,688	248,800	35,112	14.11
269,145	274,100	4,955	1.81
263,320	247,500	15,820	6.39
	Average % Difference		9.48

TABLE 27

PREDICTED VERSUS ACTUAL VALUES
(Stratification by Type of Construction, Age, and Income per Room)

Parent Sample: 43 (See Table 23)
Average Multiplier: 6.6848

Predicted	Actual	Difference	% Difference
48,933	50,000	1,067	2.13
87,838	89,000	1,162	1.31
137,493	150,000	12,507	8.34
168,457	160,000	8,457	5.29
157,694	160,000	2,306	1.44
227,818	209,000	18,818	9.00
219,261	209,000	10,261	4.91
232,631	241,000	8,369	3.47
264,718	247,500	17,218	6.96
361,380	339,225	22,155	6.53
	Average % Difference		4.94

Random Sample: 15 (See Table 23)
Average Multiplier: 6.7715

Predicted	Actual	Difference	% Difference
49,567	50,000	433	0.87
88,978	89,000	22	0.02
139,276	150,000	10,724	7.15
170,642	160,000	10,642	6.65
159,740	160,000	260	0.16
230,773	209,000	21,773	10.42
222,105	209,000	13,105	6.27
235,648	241,000	5,352	2.22
268,151	247,500	20,651	8.34
366,067	339,225	26,842	7.91
	Average % Difference		5.00

TABLE 28

PREDICTED VERSUS ACTUAL VALUES
(Stratification by Type of Construction, Age, and Number of Suites)

Parent Sample: 46 (See Table 24)
Average Multiplier: 6.7208

Predicted	Actual	Difference	% Difference
223,803	235,000	11,197	4.76
227,432	237,000	9,568	4.04
201,624	228,500	26,876	11.76
206,705	207,000	295	0.14
221,786	225,000	3,214	1.43
207,464	213,000	5,536	2.60
239,933	185,700	54,233	29.20
231,464	238,500	7,036	2.95
233,884	241,000	7,116	2.95
272,031	274,100	2,069	0.75
	Average % Difference		6.06

Random Sample: 15 (See Table 24)
Average Multiplier: 6.5199

Predicted	Actual	Difference	% Difference
217,113	235,000	17,887	7.61
220,633	237,000	16,367	6.91
195,597	228,500	32,903	14.40
200,526	207,000	6,474	3.13
215,157	225,000	9,843	4.37
201,263	213,000	11,737	5.51
232,760	185,700	47,060	25.34
224,545	238,500	13,955	5.85
226,893	241,000	14,107	5.85
263,899	274,100	10,201	3.72
	Average % Difference		8.27

on the same data.

Table 23 (Regression Equation)	Table 27 (Average Multiplier)
6.86%	4.94%
6.84%	5.00%

Table 24 (Regression Equation)	Table 28 (Average Multiplier)
5.23%	6.06%
6.94%	8.27%

In the first instance, the average multiplier proves to be slightly more accurate (on the average) than the regression equation. Use of the average multiplier results in only one of the predicted values falling outside the 10.00% limit while use of the regression equation results in three of the predicted values falling outside the 10.00% limit. In the second instance, the average multiplier proves to be slightly less accurate (on the average) than the regression equation. Use of the average multiplier results in only two of the predicted values falling outside the 10.00% limit while use of the regression equation results in three of the predicted values falling outside the 10.00% limit. It should be noted, however, that the two "wide" predictions resulting from use of the average multiplier are somewhat more extreme than the three "wide" predictions resulting from use of the regression equation.

Thus, although the tests are less than extensive and the results cannot be considered conclusive, the analysis of data in Tables 25, 26, 27, and 28 appears to indicate that there

is no really significant pattern in the difference of results obtained by the two methods--i.e. the regression equation and the average multiplier--and accuracy remains well beyond 10.00%.[33]

THE MEDIAN MULTIPLIER

In deference to the many criticisms of straight averaging, and the apparent support accorded the use of the median, the analysis of Tables 27 and 28 is replicated in this section. The objective is that of discovering whether or not use of the median multiplier provides results which are notably different from those of the average multiplier and, coincidentally, from those of the regression equation.

Table 23 (Regression Equation)	Table 27 (Average Multiplier)	Table 29 (Median Multiplier)
6.86%	4.94%	4.93%
6.84%	5.00%	4.96%

Table 24 (Regression Equation)	Table 28 (Average Multiplier)	Table 30 (Median Multiplier)
5.23%	6.06%	5.47%
6.94%	8.27%	8.26%

As is indicated in the above tabulation, results obtained through use of the median multiplier are highly similar to those obtained through use of the average multiplier. Not only are the average %

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[33]While the average % difference may be less than 10.00%, the fact that one or more of the 10 predicted values differs from the actual by more than 10.00% renders the method inaccurate to within ±10.00%.

TABLE 29

PREDICTED VERSUS ACTUAL VALUES
(Stratification by Type of Construction, Age, and Income per Room)

Parent Sample: 43 (See Table 27)
Median Multiplier: 6.670

Predicted	Actual	Difference	% Difference
48,824	50,000	1,176	2.35
87,644	89,000	1,356	1.52
137,189	150,000	12,811	8.54
168,084	160,000	8,084	5.05
157,345	160,000	2,655	1.66
227,314	209,000	18,314	8.76
218,776	209,000	9,776	4.68
232,116	241,000	8,884	3.69
264,132	247,500	16,632	6.72
360,580	339,225	21,355	6.30
	Average % Difference		4.93

Random Sample: 15 (See Table 27)
Median Multiplier: 6.710

Predicted	Actual	Difference	% Difference
49,117	50,000	883	1.77
88,169	89,000	831	0.93
138,011	150,000	11,989	7.99
169,092	160,000	9,092	5.68
158,289	160,000	1,711	1.07
228,677	209,000	19,677	9.41
220,088	209,000	11,088	5.31
233,508	241,000	7,492	3.11
265,716	247,500	18,216	7.36
362,743	339,225	23,518	6.93
	Average % Difference		4.96

TABLE 30

PREDICTED VERSUS ACTUAL VALUES
(Stratification by Type of Construction, Age, and Number of Suites)

Parent Sample: 46 (See Table 28)
Median Multiplier: 6.802

Predicted	Actual	Difference	% Difference
226,507	235,000	8,493	3.61
230,180	237,000	6,820	2.88
204,060	228,500	24,440	10.70
209,202	207,000	2,202	1.06
224,466	225,000	534	0.24
209,971	213,000	3,029	1.42
242,831	185,700	57,131	30.77
234,261	238,500	4,239	1.78
236,710	241,000	4,290	1.78
275,318	274,100	1,218	0.44
	Average % Difference		5.47

Random Sample: 15 (See Table 28)
Median Multiplier: 6.521

Predicted	Actual	Difference	% Difference
217,149	235,000	17,851	7.60
220,671	237,000	16,329	6.89
195,630	228,500	32,870	14.39
200,560	207,000	6,440	3.11
215,193	225,000	9,807	4.36
201,297	213,000	11,703	5.49
232,800	185,700	47,100	25.36
224,583	238,500	13,917	5.84
226,931	241,000	14,069	5.84
263,944	274,100	10,156	3.71
	Average % Difference		8.26

differences considerably alike, but the number of predicted values which differ by more than 10.00% from the actual values is unchanged.

THE WHITE PAPER EFFECT

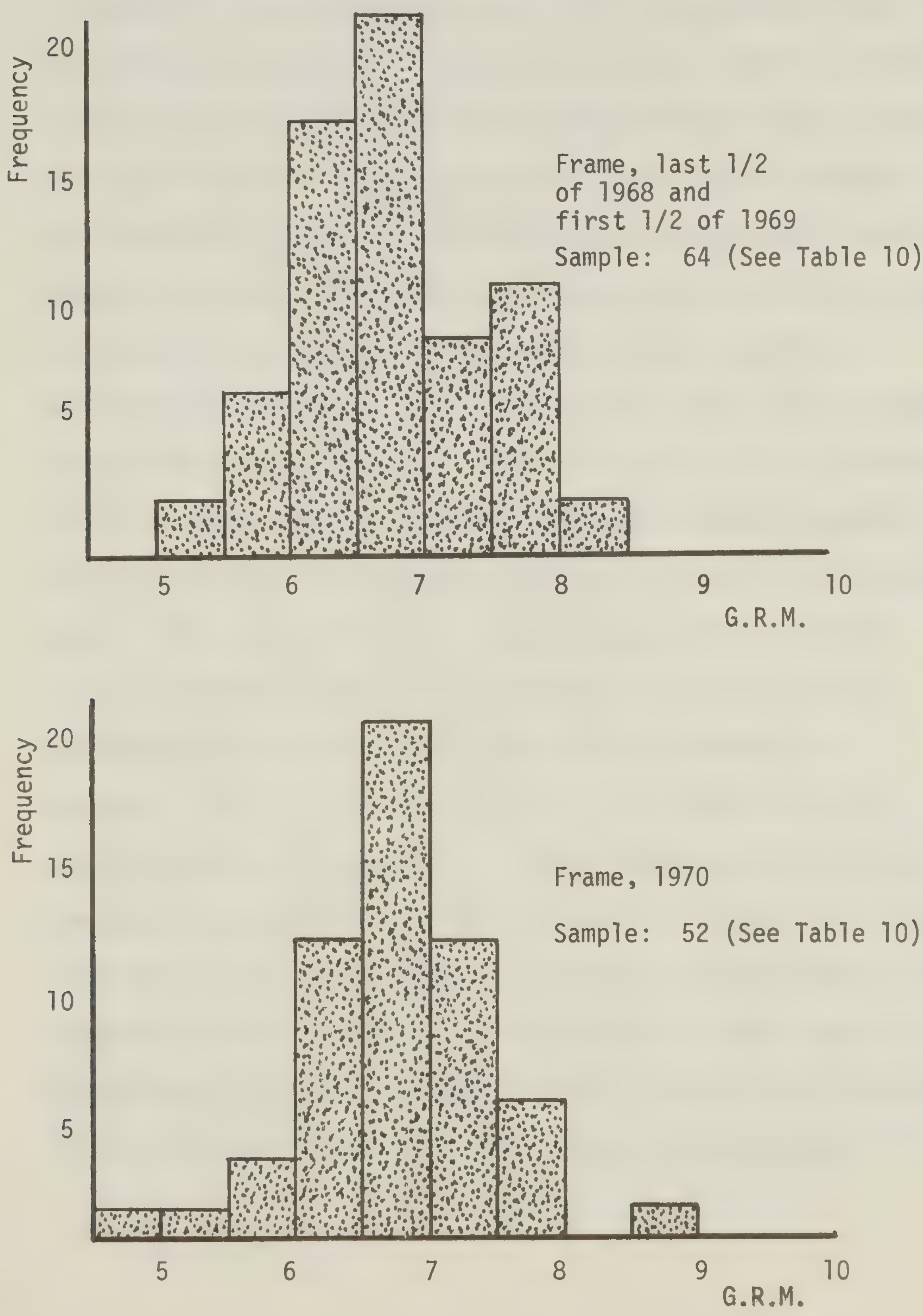
In respect of the earlier noted observations of the possible effect of the 1969 white paper on tax reform, it may be useful to examine the general level of the gross rent multipliers which arise from the before and after transactions considered in the lower portion of Table 10.

As is shown in Figure 1, the gross rent multipliers which result from the after transactions yield a frequency distribution which much more closely approximates a normal (bell-shaped) curve than does the frequency distribution which results from the before transactions. Approximately 69% of the multipliers in the before sample are within the 6.00 to 7.50 range, while some 79% of the multipliers in the after sample are within the 6.00 to 7.50 range. Not only is the pattern tighter in the after sample, but the general level of the multipliers is lower--approximately 19% of the multipliers in the before sample exceed 7.50, while less than 12% of the multipliers in the after sample exceed 7.50.

Again, because of the fact that the after sample includes considerably larger and thus more expensive properties, it can only be assumed that the tightening of the range of multipliers and the general lowering of the level of multipliers

FIGURE 1

DISTRIBUTION OF MULTIPLIERS BEFORE AND AFTER
PROPOSALS FOR TAX REFORM



is the result, at least partly, of the proposed changes in tax legislation. It could well be, as was suggested earlier, that part, or perhaps all, of the difference between the two samples is due to the fact that the more-sophisticated investor is involved to a greater degree in the after transactions than in the before transactions. Perusal of the data of Table 4 indicates that the average selling price of the properties which make up the after sample is approximately 31% higher than the average selling price of the properties which make up the before sample. Changes in lender preferences and rising land costs (precipitated by a growing shortage of good apartment sites) encouraged apartment developers to concentrate on the larger, more economical higher-density developments in 1969 and 1970, and it is a matter of record that a larger proportion of higher-density apartment buildings was built for resale in 1969 and 1970 than in previous years. Thus, the prevalence of the large-property purchaser in the after market was to a large extent a function of the changing nature of the supply of properties available for purchase. Then, if one accepts the premise that, in general, the large-property purchaser is a more-sophisticated investor, the question becomes: To what extent is the "tightening" and "lowering" of the multipliers the result of a market which is more saturated with sophisticated purchasers and to what extent is it due to the so-called white paper effect? Obviously no definitive answer is possible, but since the large-property purchaser is

most often the higher-tax bracket investor with the most to lose from implementation of the proposals for tax reform it would seem reasonable to assume that his purchases in the after market would reflect harder bargaining in an initial attempt to hedge against the possibility of such reform proposals becoming law.

An additional test of the white paper effect is attempted in Tables 31 and 32. A relatively homogeneous group of properties (stratified at four levels) is selected from the before transactions. The regression equation produced by this group of properties is used to predict the values of similar properties in the after transactions. The predicted values are then compared with the actual selling prices and the average % difference is calculated.

It was found that the 157 properties contained 33 which were of frame construction, 5 years old or newer, less than 18 suites in size, and which had been sold in the last 1/2 of 1968 or the first 1/2 of 1969. These 33 properties were used to produce a regression equation which was applied to the gross income of 16 similar properties sold in 1970. The resulting predicted values were then compared to the actual selling prices as is shown in Table 31. As may be seen, the average % difference is 6.54%. The predicted value exceeds the actual selling price in 6 cases and is smaller than the actual selling price in 10 cases.

In Table 32, the predicted values were calculated via a regression equation which was produced by 19 properties which were of frame construction, 5 years old or newer, earning $363 (or more) gross rent per room, and which had been sold in the

TABLE 31

PREDICTED VERSUS ACTUAL VALUES
(Stratification by Type of Construction, Age, Number of Suites, and Date*)

Predicted	Actual	Difference	% Difference
53,110	49,400	+ 3,710	7.51
67,255	56,000	+ 11,255	20.10
78,295	70,000	+ 8,295	11.85
113,830	116,000	- 2,170	1.87
123,490	120,000	+ 3,490	2.91
158,128	165,000	- 6,872	4.16
105,550	121,500	- 15,950	13.13
131,080	144,853	- 13,773	9.51
130,620	130,000	+ 620	0.48
138,670	151,500	- 12,830	8.47
152,470	141,600	+ 10,870	7.68
149,365	159,000	- 9,635	6.06
154,885	156,000	- 1,115	0.71
154,540	160,000	- 5,460	3.41
147,295	151,000	- 3,705	2.45
160,681	168,000	- 7,319	4.36
	Average % Difference		6.54

*Frame, 5 years old or newer, less than 18 suites, 1970

Regression Equation produced by:

Frame, 5 years old or newer, less than 18 suites, last 1/2 1968 and first 1/2 1969

N = 33 $\bar{X}$ = 16,387

Yx = 14,470 + 5.750X = 108,696

Sy·x = (1.96)(8,317) = 16,301

Confidence Interval = ±15.00%

R^2 = 0.9610 F-value = 764.37

TABLE 32

PREDICTED VERSUS ACTUAL VALUES
(Stratification by Type of Construction, Age, Income per Room, and Date*)

Predicted	Actual	Difference	% Difference
82,183	70,000	+ 12,183	17.40
129,996	120,000	+ 9,996	8.33
146,055	151,500	- 5,445	3.59
163,209	156,000	+ 7,209	4.62
155,179	151,000	+ 4.179	2.77
169,341	168,000	+ 1,341	0.80
187,663	171,000	+ 16,663	9.74
220,511	237,000	- 16,489	6.96
211,630	230,000	- 18,370	7.99
221,971	209,000	+ 12,971	6.21
224,890	239,500	- 14,609	6.10
228,905	229,300	- 395	0.17
226,350	241,000	- 14,650	6.08
209,561	215,000	- 5,439	2.53
210,145	248,800	- 38,655	15.54
260,878	274,100	- 13,222	4.82
255,549	247,500	+ 8,049	3.25
270,148	260,000	+ 10,148	3.90
335,844	341,725	- 5,881	1.72
401,540	353,100	+ 48,440	13.72
342,049	397,000	- 54,951	13.84
444,231	471,000	- 26,769	5.68
	Average % Difference		6.63

*Frame, 5 years old or newer, $363 and over, 1970

Regression Equation produced by:

Frame, 5 years old or newer, $363 and over, last 1/2 1968 and first 1/2 1969

$N = 19$ $\bar{X} = 26,670$

$Yx = 14,662 + 6.083X = 176,888$

$Sy \cdot x = (2.110)(14,103) = 29,757$

Confidence Interval = $\pm 16.82\%$

$R^2 = 0.9720$ F-value = 590.54

last 1/2 of 1968 or the first 1/2 of 1969. Twenty-two similar properties were found to have sold in 1970, and the values predicted for these 22 were compared with the actual selling prices. As may be seen, the average % difference is 6.63%. The predicted value exceeds the actual selling price in 10 cases and is smaller than the actual selling price in 12 cases.

Since the predicted values are not predominantly high (or low) when compared with the actual selling prices, and because the average % difference produced by each of the tests is roughly in line with the average % differences produced in earlier tests, it must be concluded that there is no indication that the findings contained in Figure 1 are explained by a changed tax environment. In fact, there appears to be no indication at all that the tax reform proposals caused any recognizable change in apartment property values to the end of 1970.

CHAPTER V

SUMMARY AND CONCLUSIONS

SUMMARY

The prime concern of this study is that of determining whether or not the gross rent multiplier may be accepted as a satisfactorily reliable appraisal device.

A survey of pertinent and available literature reveals that the gross rent multiplier is both revered and scorned. It is considered either satisfactorily accurate or useful under special (or limited) conditions or totally unreliable.

Tests of the reliability or usefulness of the gross rent multiplier appear to be limited to assessment of its accuracy; there appears to be no empirical evidence to show that it is in fact widely used (and thus acceptable to) investors. Some tests of its accuracy, together with modest evidence of its use by appraisers, are held out as evidence that it is relied upon by investors.

Information, gathered in respect of apartment property sales which occurred from mid-1968 to the end of 1970 in the Edmonton area, is analysed in this study in order to determine (a) the strength of relationship between gross incomes and selling prices and (b) the degree of predictive accuracy

attached to use of the gross rent multiplier. The tests consist of simple linear regression and correlation analysis and comparison of predicted and actual values through the application of regression equations, average multipliers, and median multipliers.

Attempt is made also to determine the effect, if any, of major proposals for tax reform, announced in November of 1969, on general levels of value and on the predictive ability of the gross rent multiplier.

CONCLUSIONS

With the exception of the results obtained from analysis of one data grouping[1], the tests indicate that the gross rent multiplier cannot be considered satisfactorily reliable as an appraisal device in the Edmonton area. Regression analysis, applied to various stratifications of data, fails to yield an equation which can be considered accurate to within 10%. The narrowest confidence interval obtained, at the 95% level of confidence, is ±10.73%.[2] Confidence intervals range, in general, from ±14.00% to ±22.00%.

Stratification of the data, into increasingly homogeneous groupings, is expected to be reflected in increasingly narrower confidence intervals. In many instances, however, the

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[1]See first test of Table 27.

[2]It should be noted that the regression equation is accompanied in this instance by a found F-value which fails to exceed the table F-value and by an R^2 which is equal to only 0.3456.

sample becomes extremely small, or non-existent, before notable narrowing can be achieved (if it indeed would be) and in other instances, where sample size is not greatly decreased by stratification, narrowing simply does not occur (at least not consistently and not to any significant degree).

Tests of predictive reliability (via the regression equation, average multiplier, and median multiplier) indicate that employment of the relationship between gross incomes and selling prices as an appraisal device might yield predicted values which are within 10% of actual values--but this would be true only "on the average" and the appraiser would have to be aware that roughly 10% to 50% of his predicted values could differ from actual selling prices by more than 10.00%.

Ratcliff, in his study of several relatively small samples of income-producing realty, claimed that (a) 80 to 96 per cent of the variation in price was explained by the variation in income[3], (b) measures of confidence indicated that the relationships underlying the equations were close to the measured relationships, and (c) the average deviation[4] was approximately 2% of actual price.

Results of the Farish study indicate that (a) the value of R^2 is predominantly over 90% (although it ranges from 0.5746

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[3]Richard U. Ratcliff, Current Practices in Income Property Appraisal--A Critique (Berkeley: University of California, 1967). A summary of regression analysis contained in pages 55 to 60 of Ratcliff's study shows a range of values for R^2 of from 0.666 to 0.964.

[4]The ratio of the mean deviation to the mean actual price.

to 0.9995), and (b) confidence intervals range from ±29.22% (for the entire sample) to ±9.27% (best test result).[5] Comparison of predicted selling prices with actual selling prices is shown to provide an average % difference which ranges from approximately 3% to 7.5% in the case of regression equations which are obtained from two-level stratification and from approximately 3% to 9.5% in the case of regression equations which are obtained from data stratification at three levels.[6]

Thus, while Ratcliff and Farish found that their analyses of data indicated relatively strong correlation of gross rents and selling prices, and that tests of the predictive reliability of the gross rent multiplier appeared to indicate acceptable accuracy, the results of these studies are not dramatically unlike the results produced by analysis of the Edmonton data. It may be that the results reported by Ratcliff and Farish are not as statistically significant as is intimated.

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[5]W.G. Farish, "The Use of Direct Conversion Ratios and the Selection of Capitalization Rates in Residential Income Property Appraisal" (unpublished M.B.A. dissertation, University of British Columbia, 1968). The regression equation for this best test result (the only instance in which the confidence interval narrowed to within 10%) is accompanied by a found F-value which exceeds the table F-value and by R^2 which is equal to 0.9841.

[6]Farish does not show the predicted and actual values (nor the % difference), he shows only the average % difference--it is thus impossible to know the number of instances, if any, in which the predicted value differs from the actual by more than 10%. It is also important to note that the Farish study does not attempt to test data stratified at more than three levels.

Both studies are conspicuous by the absence of detail. However, on the assumption that the results are as statistically significant as reported, it is desirable to outline possible reasons for (or causes of) the difference between their findings and the findings produced by this study.

A major reason for the difference in findings may lie in the fact that the Cities from which the data are drawn are markedly dissimilar. Los Angeles and Vancouver are considerably different from Edmonton in size and character, and the real estate market in Edmonton can be expected to differ notably from that of these large, older, port cities. The difference in size of data sample and the difference in time periods examined may also help to explain variation in the findings. It should also be noted that variation in sources of data may be the cause of some difference. In respect of the latter, it is important to note that Farish's information on sales was secured:

> ... from all reliable sources, mainly realtors, appraisers, and tax assessment officers ...[7]

It might be suspected that one of the reasons for the better relationship between the gross rents and selling prices of the Vancouver data stems from a tendency on the part of brokers, appraisers, and assessors to provide gross rent estimates which are in line with their awareness of rental trends in the market rather than their knowledge of the actual rents earned by each of the

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[7]Richard U. Ratcliff, "Don't Underrate the Gross Income Multiplier," The Appraisal Journal, XXXIX, No. 2 (1971), p. 265.

properties at the time of sale. Owners are generally looking for the highest price obtainable, the best mortgage available, and the lowest assessment possible. It is therefore in their interest to mislead or misguide realtors, appraisers, and assessors under most circumstances. Of the 157 gross rent estimates gathered for the Edmonton study, 149 were provided by the vendor or purchaser, or both.[8] However, although it may be reasonable to suspect that at least the gross rent information in the Edmonton data is more reliable than the gross rent information in the Vancouver data, it is clear, and admitted, that nothing contained in this study can be taken as a guarantee that the vendors and purchasers have provided the writer with figures which are more correct than would have been obtained from realtors, appraisers, and assessors.

Tests in respect of the comparative reliability of the regression equation and the average multiplier tend to bear out the findings of the Farish study--namely, that the average multiplier provides estimates which are as close, on the average, to the predicted values as those provided by the regression equation. The significance of this is that to the extent that the relationship between gross incomes and rents is useful in real property appraisal the appraiser need not understand regression analysis, nor employ a computer program, in order to take full

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[8]In addition to provision of gross income data, each of the replies could be taken as the respondent's confirmation that a bona fide sale had taken place.

advantage of the usefulness of the relationship. Additional tests indicate that the median multiplier can probably not be expected to produce results significantly different from those obtained via the average multiplier.

As well, both the regression analysis and the tests of predictive accuracy indicate generally that, within the same grouping, small samples provide equations and multipliers of about the same reliability as those which are provided by large samples. Extremely small samples, however, are likely to provide statistically meaningless results--chiefly because of the effect of the t-values on the width of the confidence interval.

A further noteworthy conclusion which might be drawn from the results of this study is the indication that while some of the pertinent and available literature may imply investor reliance on the gross rent multiplier it does not demonstrate the logic of such reliance for anything but a preliminary guide to (or rough check on) the value of income-producing realty. Neither does it provide evidence to show that investors are, in fact, using the gross rent multiplier (or, for that matter, any other direct conversion ratio) to any significant degree. To the contrary, it suggests that more than casual reliance on the gross rent multiplier would be illogical. Moreover, this study's survey of pertinent and available literature indicates an apparent lack of understanding in respect of the ways in which

investors actually make their decisions to buy and to pay a certain price. This can only be taken as indication that what is required now is not so much a replication of this study in another urban area but a study (or enquiry of some sort) which is devoted to discovery of major variations in the financial and behavioural characteristics of investors. Perhaps such a study would indicate, in reflection of Dorau's thesis that a multiplicity of factors other than income are known to influence value, that the appraiser should be concerned more with stratification and analysis of the investor than with stratification and analysis of the income-producing realty.[9] As Ratcliff has stated:

> ... perhaps most important of all is the appraiser's attempt to understand the calculus of investors ... so that he may translate its (a property's) income-productivity potential into a capital value figure in the same manner and with the same results as do real-life investors ... there is no predictive value in employing sophisticated methods of investment analysis, such as are suggested in the conventional income approach presented in most textbooks, if real-life buyers do not use these methods. The appraiser must discover what methods actual investors do, in fact, use and must himself use these methods whether or not he considers them correct or adequate.[10]

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[9]To this end, despite the obvious difficulty which would be encountered in quantifying some of the non-(annual)income factors, multiple regression analysis might be found more useful than simple linear regression analysis.

[10]Richard U. Ratcliff, Modern Real Estate Valuation (Madison: Democrat Press, 1965), p. 167.

Lastly, the results of this study indicate that the proposals for tax reform announced in November of 1969 did not cause any recognizable change in apartment property values through 1970.

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APPENDIX

The data contained in the following pages consists of:

the questionnaire,
the 157 calculated gross rent multipliers,
discussion of statistical methods and formulae,
regression and correlation statistics,
a large-sample comparison of predicted and actual values,
a sample of the computer program input,
a sample of the computer program output, and
a map of the districts.

With respect to the regression and correlation statistics, the table F-value is shown in brackets beside the found F-value only in those instances where the found F-value fails to equal or exceed the table F-value.

FACULTY OF BUSINESS ADMINISTRATION AND COMMERCE

THE UNIVERSITY OF ALBERTA
EDMONTON 7, CANADA

I am currently completing research in the Masters program in Business Administration at the University of Alberta. The object of the study is to determine the usefulness of "conversion ratios" to persons who are appraising or estimating the value of apartment properties. In particular, we are testing the predicting ability of the gross rent multiplier (selling price ÷ gross annual income) in 392 apartment building sales which occurred in the City of Edmonton during the past 2½ years.

It is our understanding that you are either the purchaser or seller of a _____ suite apartment building located at ______________________________. We would appreciate your assistance in estimating the actual gross income of the property at the date of sale. Data will be treated as confidential.

Would you please fill in either of the blanks below and return this letter in the enclosed envelope on or before March 22nd, 1971.

$________________ Actual Monthly Gross Income at time of sale (assuming full occupancy plus any parking rental)

$________________ Gross Yearly Income (assuming full occupancy plus parking rental) as indicated by rentals at time of sale.

Your assistance is sincerely appreciated.

Donald N. Liteplo
Candidate for MBA Degree,
University of Alberta.

CALCULATED GROSS RENT MULTIPLIERS

Identification No. of Property	No. of Suites	Selling Price	Gross Rent	Gross Rent Multiplier
001	4	44,000	5,280	8.333
002	4	54,000	6,780	7.965
003	4	66,000	10,620	6.215
004	4	50,000	7,320	6.831
005	4	60,000	6,300	9.524
006	4	41,000	5,652	7.254
007	4	57,000	8,400	6.786
008	4	41,500	5,220	7.950
009	4	45,000	5,640	7.979
010	4	49,400	6,720	7.351
011	4	59,658	7,800	7.648
012	4	69,000	8,760	7.877
013	4	50,000	6,720	7.440
014	4	56,600	7,652	7.397
015	6	59,000	8,400	7.024
016	6	40,500	6,250	6.480
017	6	56,000	9,180	6.100
018	6	63,500	11,040	5.752
019	6	45,000	7,500	6.000
020	6	56,400	7,080	7.966
021	6	41,500	5,288	7.848
022	6	46,500	6,840	6.798
023	6	52,000	8,400	6.190
024	6	54,075	7,680	7.041

CALCULATED GROSS RENT MULTIPLIERS (Continued)

Identification No. of Property	No. of Suites	Selling Price	Gross Rent	Gross Rent Multiplier
025	6	49,274	7,652	6.439
026	6	50,000	7,500	6.667
027	6	70,000	11,100	6.306
028	8	82,000	12,720	6.447
029	9	93,500	14,460	6.466
030	9	80,863	10,260	7.881
031	9	100,000	12,960	7.716
032	9	105,500	13,900	7.590
033	9	89,000	13,140	6.773
034	9	80,000	16,200	4.938
035	9	92,500	11,400	8.114
036	9	82,000	12,600	6.508
037	9	102,000	13,080	7.798
038	9	104,000	10,980	9.472
039	9	78,000	11,220	6.952
040	9	91,384	11,676	7.827
041	10	102,000	14,880	6.855
042	11	108,200	16,416	6.591
043	12	102,000	15,024	6.789
044	12	116,000	17,280	6.713
045	12	111,400	16,920	6.584
046	12	120,000	18,960	6.329
047	12	165,000	24,984	6.604
048	12	109,800	18,660	5.884

CALCULATED GROSS RENT MULTIPLIERS (Continued)

Identification No. of Property	No. of Suites	Selling Price	Gross Rent	Gross Rent Multiplier
049	12	175,000	13,920	12.572
050	12	208,000	17,640	11.791
051	12	105,000	16,260	6.458
052	12	139,470	17,694	7.882
053	12	121,500	15,840	7.670
054	12	129,600	18,420	7.036
055	12	96,600	14,000	6.900
056	12	96,615	16,260	5.942
057	12	130,000	20,976	6.198
058	12	133,600	17,100	7.813
059	12	144,853	20,280	7.143
060	12	128,000	20,940	6.113
061	12	197,000	16,200	12.160
062	12	123,500	20,580	6.001
063	12	111,000	16,068	6.908
064	12	97,000	15,024	6.456
065	12	102,500	17,760	5.771
066	12	101,000	14,640	6.899
067	12	131,200	19,200	6.833
068	12	130,000	18,720	6.944
069	14	130,000	20,200	6.436
070	14	151,500	21,600	7.014
071	14	150,000	20,568	7.293
072	14	139,000	22,080	6.295

CALCULATED GROSS RENT MULTIPLIERS (Continued)

Identification No. of Property	No. of Suites	Selling Price	Gross Rent	Gross Rent Multiplier
073	14	144,000	22,080	6.522
074	14	145,000	21,588	6.717
075	14	127,000	20,160	6.300
076	15	153,000	24,180	6.328
077	15	141,600	24,000	5.900
078	15	164,000	24,900	6.586
079	15	159,000	23,460	6.777
080	15	156,000	24,420	6.388
081	15	155,000	21,960	7.058
082	15	119,500	22,680	5.269
083	15	144,500	23,820	6.066
084	15	124,000	19,380	6.398
085	15	160,000	25,200	6.349
086	15	148,000	25,200	5.873
087	15	154,500	24,120	6.405
088	15	160,000	24,360	6.568
089	15	151,000	23,100	6.537
090	15	157,000	23,100	6.797
091	15	144,041	23,280	6.187
092	15	178,000	24,096	7.387
093	15	153,975	23,680	6.502
094	16	152,000	23,400	6.496
095	16	166,000	25,920	6.404
096	16	168,000	25,428	6.607

CALCULATED GROSS RENT MULTIPLIERS (Continued)

Identification No. of Property	No. of Suites	Selling Price	Gross Rent	Gross Rent Multiplier
097	16	180,000	30,432	5.915
098	17	160,000	23,590	6.783
099	18	162,000	27,000	6.000
100	18	161,000	28,080	5.734
101	18	170,000	28,800	5.903
102	18	235,000	33,300	7.057
103	18	171,000	28,440	6.013
104	18	225,000	30,900	7.282
105	18	187,450	26,460	7.084
106	18	210,000	31,920	6.579
107	18	271,200	38,400	7.063
108	18	237,000	33,840	7.004
109	18	198,500	30,060	6.603
110	19	230,000	32,380	7.103
111	19	201,500	30,900	6.521
112	19	181,000	27,060	6.689
113	20	200,000	29,460	6.789
114	20	228,500	30,000	7.617
115	20	209,000	34,080	6.133
116	20	222,000	33,960	6.537
117	20	223,000	28,800	7.743
118	20	239,500	34,560	6.930

CALCULATED GROSS RENT MULTIPLIERS (Continued)

Identification No. of Property	No. of Suites	Selling Price	Gross Rent	Gross Rent Multiplier
119	20	207,000	30,756	6.730
120	20	217,338	31,200	6.966
121	20	205,000	30,000	6.833
122	20	228,000	33,600	6.786
123	20	233,500	29,952	7.796
124	20	225,000	33,000	6.818
125	20	229,300	35,220	6.511
126	20	198,000	25,040	7.907
127	20	227,000	32,000	7.094
128	20	213,000	30,869	6.900
129	21	209,000	32,800	6.372
130	21	230,000	34,080	6.749
131	21	200,000	28,560	7.003
132	21	186,000	35,700	5.210
133	21	195,000	32,400	6.019
134	21	185,700	35,700	5.202
135	21	220,000	33,860	6.497
136	21	203,000	33,300	6.096
137	22	238,500	34,440	6.925
138	22	229,100	32,520	7.045
139	22	248,000	34,380	7.213
140	22	241,000	34,800	6.925
141	22	215,000	32,040	6.710

CALCULATED GROSS RENT MULTIPLIERS (Continued)

Identification No. of Property	No. of Suites	Selling Price	Gross Rent	Gross Rent Multiplier
142	22	248,800	32,136	7.742
143	23	265,000	37,900	6.992
144	23	274,100	40,476	6.772
145	23	247,500	39,600	6.250
146	24	260,000	42,000	6.190
147	24	253,500	31,500	8.048
148	26	286,000	32,571	8.781
149	27	270,000	40,510	6.665
150	30	341,725	52,800	6.472
151	33	353,100	63,600	5.552
152	33	339,225	54,060	6.275
153	33	397,000	53,820	7.376
154	33	331,000	55,620	5.951
155	36	534,000	77,160	6.921
156	42	471,000	70,618	6.670
157	45	611,000	81,480	7.499

STATISTICAL METHODS AND FORMULAE

Correlation analysis is a means of measuring the relationship or association that may exist between the values of two variables.[1] The strength of relationship may be indicated in a variety of ways. The more often used measures include the coefficient of determination, R^2, and the value used to indicate the significance of the regression coefficient, F.

R^2, which varies from zero to one, is the proportion of the variance in the dependent variable accounted for by the regression equation.[2] The nearer R^2 is to one, the stronger is the indication that the independent variable may be helpful in making a useful prediction of the dependent variable. The R^2 is calculated by the following formula:

$$R^2 = \frac{\left(\Sigma XY - \frac{\Sigma X \Sigma Y}{N}\right)^2}{\left(\Sigma X^2 - \frac{(\Sigma X)^2}{N}\right)\left(\Sigma Y^2 - \frac{(\Sigma Y)^2}{N}\right)}$$

where the Y values are those of the dependent variable, the X values are those of the independent variable, and N is the sample size.

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[1] Allen L. Edwards, Statistical Analysis (New York: Holt, Rinehart, and Winston, Inc., 1969), p. 66.

[2] Norman Nie, Dale H. Bent, and C. Hadlai Hull, Statistical Package for the Social Sciences (New York: McGraw-Hill Book Company, 1970), p. 185.

The F measures the significance of the regression equation representing more than mere chance.[3] The F-level is interpreted through the use of tables which give the probability of obtaining a given F by mere chance. If the obtained value F has a small probability, then either a rare or improbable event has occurred by chance, or the variables are highly dependent, one upon the other, and the regression equation may be said to be significant.[4] The table values of F examined for purposes of this study include values which would be expected to be equalled or exceeded with a probability of 0.05. If the found F-value is, in fact, equal to or greater than the table value, the regression coefficient (b) may be said to be significantly different from zero and the regression equation may be accepted as being significant (i.e. not likely the result of mere chance). The F-value is calculated by the following formula:

$$F(1, N-2) = \frac{\sum(Yx - \bar{Y})^2}{(Sy \cdot x)^2}$$

where Yx is the estimated or computed line value of Y given X, $\bar{Y}$ is the mean selling price (or capital value), and $Sy \cdot x$ is the estimate of standard error $\sqrt{\frac{(Y - Yx)^2}{N-2}}$.

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[3] Ibid.

[4] Edwards, Statistical Analysis, p. 145.

Regression analysis is a means of making a prediction about the value of one variable given that some particular value of the other variable has been obtained.[5]

Regression, as employed in this study, is both simple and linear. It is simple because only two variables were tested for strength of relationship. Many variables were considered, but the method of measurement of their effect was via stratification so that it was neither necessary nor desirable to experiment with multiple-regression analysis. The analysis is based on the assumption of linearity because correlation analysis can provide an adequate measure of the relationship between two variables only when they are linearly related.[6] This assumption is inherent in the SPSS computer subprogram which is utilized in this study. The description of the subprogram includes notation of the assumption that the dependent variable changes by a constant amount when the independent variable changes by a given absolute amount.[7] When the two variables were subjected to initial examination by way of plotting their values on a graph, they were found, in fact, to be in linear relationship.

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[5]Henry E. Klugh, Statistics: The Essentials for Research (New York: John Wiley and Sons, Inc., 1970), p. 66.

[6]Edwards, Statistical Analysis, p. 74.

[7]Nie, Bent, and Hull, Statistical Package for the Social Sciences, Passim, chap. XV.

The coefficient of determination, R^2, is equal to 1.00 where plotting of the two values indicates perfect linear relationship. In those cases where the trend is linear but the relationship is not "perfectly" linear, a straight line (the regression line) can be fitted to the plotted values by means of the least squares method. This method minimizes the sum of the squared deviations from the line and thus yields a line which fits the plotted values better than would any other.[8] It thereby provides the best possible regression equation

$$Yx = a + bX$$

where Yx is the estimated or computed line value of Y given X, (a) is the estimate of the constant which represents the value of Y when X is zero, (b) is the estimate of the coefficient of regression which shows the amount of change in Y which is associated with a one-unit change in X, and X is the independent variable.[9]

The (a) and (b) values of a straight line fitted by the least squares method are obtained by solving the following two normal equations simultaneously:[10]

$$\Sigma Y = Na + b\Sigma X$$
$$\Sigma XY = a\Sigma X + b\Sigma X^2$$

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[8]Edwards, Statistical Analysis, p. 77.

[9]Boris Parl, Basic Statistics (Garden City, New York: Doubleday and Company, Inc., 1967), p. 227.

[10]Ibid.

In order to gauge the usefulness of a given regression equation to the appraiser, the width of the confidence interval must be calculated.

The process of examining a sample in order to predict or estimate characteristics of the population from which the sample was drawn is termed "statistical inference."[11] The sample is assumed to be representative of the population, and knowledge of the characteristics of the sample is assumed to provide knowledge of the characteristics of the population. This is because the sample is expected to be normally distributed within the population (i.e. subject to graphical representation by a symmetrical and bell-shaped curve). Extensive study has shown that a great many distributions of psychological and biological variables are described by such a curve.[12] The reasons for such normality are considered to be relatively unimportant. What is important is the fact that the interesting mathematical properties of the curve can be put to good use in the examination of real distributions (e.g. the 157 apartment property sales which make up the raw data of this study).[13] However, the sample is seldom, if ever, perfectly representative of the population, and statistical inference is considered always to be subject to error. Recognition

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[11]Edwards, Statistical Analysis, p. 8.

[12]Klugh, Statistics: The Essentials for Research, p. 49.

[13]Ibid.

of the error is generally manifested via expression of some "degree of confidence" in the statistical inference.

The degree of confidence is reflected in measurement of the aforementioned width of the confidence interval. The confidence interval is the product of the estimate of the standard error of estimate (Sy·x) and the standard normal deviate which corresponds to the desired confidence level.[14]

The width of the confidence interval may be calculated at any level. The 95 per cent confidence level is the most frequently used. Normal curve tables show the standard normal deviate, at the 95 per cent level of confidence, to be 1.96. If the standard error of estimate is 21,073, as is indicated in the first test shown on Table 8, the width of the confidence interval, at the 95 per cent level of confidence, is found to be:

$$(1.96)(21{,}073) = \pm 41{,}302.$$

This confidence interval enables the statistician or researcher to specify a range of values whithin which an estimate (obtained via the regression equation) will have some specified probability of occurrence.[15] In this instance, the probability is only 5 per cent, or one chance in twenty, that the selling price which is estimated by the regression equation Yx = 5,323 + 6.560X will vary from the actual selling price by more than $41,302.

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[14] Ibid., p. 140.

[15] Ibid.

Another way of stating this is to say that if the researcher were to use the regression equation to predict the selling price of 100 apartment properties, he would be confident that in at least 95 cases the actual selling price would not exceed the predicted selling price by more than $41,302, nor would it be less than the predicted selling price by more than $41,302. Thus, the width of the confidence interval is expressed as ±$41,302.

Other, often used, levels of confidence were rejected for use in this study. The 99 per cent level permits the user to state that there is probably only one chance in one hundred that the actual selling price will fall outside of the indicated width of the confidence interval. However, in order to gain this increased confidence, the confidence interval must be considerably widened. Conversely, if narrower limits for the interval are desired, the researcher must be prepared to accept a considerable reduction in the confidence level.[16] This explains, in part at least, the earlier observation that the 95 per cent level is one of the most frequently used levels of confidence. In addition, the 95 per cent level would appear to be the most convenient and acceptable level for use in this study because it permits some comparison of results with those obtained in the Farish study.[17]

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[16] Ibid.

[17] Farish, "The Use of Direct Conversion Ratios and the Selection of Capitalization Rates in Residential Income Property Appraisal," p. 73.

When samples are small, the sample variance is found to be a biased estimator of the population variance.[18] In order to minimize the effect of such bias when dealing with samples of 31 or less, the standard normal deviate obtained from normal curve tables is discarded in favour of a factor obtained from t-distribution tables. Thus, in those instances where the regression equation resulted from analysis of a sample of less than 32 apartment properties, the t-distribution was used to calculate the width of the confidence interval. Values of t, for the appropriate number of degrees of freedom (N-2 in the regression calculation) were selected, of course, at the 95 per cent level of confidence.

In addition to the preceding discussion of the characteristics and limitations of regression and correlation analysis, it must be noted that the computed regression line, which is based on a sample of X values (gross rents), is inappropriately used in deriving estimates for Y (selling prices) where the line has been extended beyond the range of data in the sample. Such extrapolation is considered to go beyond statistical evidence as there is no assurance that a straight line relationship exists beyond the observed data range.[19] Thus, in utilizing a regression equation to estimate the selling price of a given apartment property, the user should ensure

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[18] Klugh, Statistics: The Essentials for Research, p. 206.

[19] Parl, Basic Statistics, p. 231.

that the gross rent of that property falls within the range which was used to produce the equation.

It must be noted, also, that the constant (a), which determines the height of the regression line (i.e. represents the value of Y when X is zero) and which is sometimes a negative value, is simply a mathematical necessity in the equation. It has no real meaning.[20] The (a) value is simply an extension of the linear relationship outward from the data range to the Y axis. As is noted above, linearity cannot be assumed to exist beyond the observed data range and, while common sense suggests that an apartment property would have some residual value when gross rent equals zero, the value of (a) cannot be considered a valid estimate of that value.

Lastly, it must be noted that in addition to the earlier noted standard error of estimate, there will also be significant error connected with the estimated values of (a) and (b) in those instances where the sample size is small. This error is termed the sampling error of the regression constants. However, except for very small samples, researchers usually disregard the sampling errors of the regression constants. Their inclusion considerably increases the computational burden and, except for very small samples, they are of relatively minor importance.[21] For these reasons, they are disregarded in this study.

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[20] _Ibid._, p. 227.

[21] _Ibid._, p. 242.

REGRESSION & CORRELATION STATISTICS

Properties	No.	Equation	$\bar{X}$	R^2	F-Value
All Properties	157	Yx = 5,323 + 6.560X	23,761	0.9469	2,762.46
Masonry Only	9	Yx = 591 + 6.943X	26,886	0.9812	365.08
Frame Only	148	Yx = 6,281 + 6.505X	23,571	0.9425	2,395.50
Frame Only (excluding the three extremes)	145	Yx = 2,716 + 6.583X	23,729	0.9614	3,565.47
Frame, before publication of proposals for tax reform	78	Yx = 12,372 + 6.132X	20,581	0.9300	1,009.03
Frame, after publication of proposals for tax reform	70	Yx = 2,717 + 6.704X	26,902	0.9471	1,216.69
Frame, last 1/2 of 1968 and first 1/2 of 1969	66	Yx = 14,312 + 6.060X	21,364	0.9202	737.88
Frame, 1970	52	Yx = 7,238 + 6.430X	27,972	0.9463	881.31
Frame, last 1/2 of 1968 and first 1/2 of 1969 (excluding the three extremes)	64	Yx = 9,035 + 6.188X	21,552	0.9612	1,537.51

REGRESSION & CORRELATION STATISTICS (Continued)

Properties	No.	Equation	$\bar{X}$	R^2	F-Value
Frame, 1970 (excluding the three extremes)	52	Yx = 7,238 + 6.430X	27,972	0.9463	881.31
Frame, 10 years old or newer	135	Yx = 2,839 + 6.581X	24,651	0.9588	3,094.74
Frame, 5 years old or newer	121	Yx = 3,268 + 6.570X	25,781	0.9560	2,587.13
Frame, 1 year old or newer	84	Yx = 1,249 + 6.595X	28,866	0.9494	1,539.36
Frame, 4 to 6 suites	26	Yx = 18,656 + 4.581X	7,374	0.6754	49.94
Frame, 4 to 11 suites	41	Yx = 9,180 + 6.135X	9,454	0.8276	187.25
Frame, 12 suites	19	Yx = 15,510 + 5.689X	17,965	0.7714	57.36
Frame, 12 to 16 suites	46	Yx = 11,529 + 5.949X	21,037	0.8265	209.64
Frame, 18 to 20 suites	30	Yx = 1,060 + 6.789X	31,001	0.6180	45.30
Frame, 18 to 24 suites	48	Yx = 39,945 + 5.476X	32,431	0.5302	51.92

REGRESSION & CORRELATION STATISTICS (Continued)

Properties	No.	Equation	$\bar{X}$	R^2	F-Value
Frame, less than 18 suites	88	Yx = 10,176 + 6.023X	15,670	0.9519	1,700.54
Frame, more than 17 suites	57	Yx = 5,643 + 6.557X	36,172	0.8908	448.42
Frame, Delton	28	Yx = - 34 + 6.528X	20,588	0.9366	383.81
Frame, Jasper Place	21	Yx = 6,387 + 6.196X	23,665	0.9397	296.39
Frame, King Edward Park	16	Yx = 4,020 + 6.846X	19,089	0.9641	376.03
Frame, Hudson Bay Reserve	23	Yx =-15,199 + 7.246X	29,973	0.9686	649.11
Frame, $312 and less	27	Yx = 1,933 + 7.115X	14,870	0.9666	723.46
Frame, $313 to $362	62	Yx = 7,517 + 6.236X	22,400	0.9288	783.25
Frame, $363 to $412	45	Yx = 11,311 + 6.264X	30,466	0.9628	1,114.07
Frame, $413 and over	11	Yx =-21,946 + 7.700X	25,406	0.9951	1,825.48
Frame, $313 to $412	107	Yx = 7,409 + 6.316X	25,792	0.9561	2,287.55

REGRESSION & CORRELATION STATISTICS (Continued)

Properties	No.	Equation	$\bar{X}$	R^2	F-Value
Frame, $363 and over	56	Yx = -1,187 + 6.710X	29,472	0.9664	1,554.49
Frame, 5 years old or newer, 4 to 6 suites	17	Yx = 22,478 + 4.261X	7,702	0.6913	33.59
Frame, 5 years old or newer, 18 to 20 suites	28	Yx = 3,586 + 6.714X	31,197	0.5944	38.11
Frame, 5 years old or newer, 18 to 24 suites	46	Yx = 42,634 + 5.399X	32,613	0.5073	45.31
Frame, 5 years old or newer, more than 17 suites	55	Yx = 5,724 + 6.556X	36,460	0.8887	423.26
Frame, 5 years old or newer, Delton	28	Yx =-33,903 + 6.528X	20,588	0.9366	383.81
Frame, 5 years old or newer, Jasper Place	17	Yx = 16,744 + 5.897X	26,950	0.9135	158.50
Frame, 5 years old or newer, King Edward Park	15	Yx = 5,284 + 6.800X	19,862	0.9607	317.46
Frame, 5 years old or newer, Hudson Bay Reserve	18	Yx =-22,314 + 7.405X	32,995	0.9676	477.29

REGRESSION & CORRELATION STATISTICS (Continued)

Properties	No.	Equation	$\bar{X}$	R^2	F-Value
Frame, 5 years old or newer, \$363 to \$412	43	Yx = 12,201 + 6.241X	31,385	0.9595	972.32
Frame, 5 years old or newer, \$313 to \$412	96	Yx = 8,435 + 6.284X	26,882	0.9520	1,863.08
Frame, 5 years old or newer, \$363 and over	52	Yx = 106 + 6.681X	30,855	0.9639	1,336.41
Frame, 5 years old or newer, 18 to 24 suites	17	Yx = -2,545 + 6.768X	32,125	0.6260	25.11
Frame, 5 years old or newer, Hudson Bay Reserve	8	Yx =-33,819 + 7.794X	35,892	0.9834	355.38
Frame, 5 years old or newer, \$363 and over	23	Yx = -8,361 + 7.082X	26,858	0.9753	830.39
Frame, 5 years old or newer, \$313 to \$412	9(10%)	Yx = 14,918 + 6.137X	20,685	0.9777	306.55
	18(20%)	Yx = 9,108 + 6.361X	22,361	0.9761	653.09
	29(30%)	Yx = 13,343 + 6.020X	24,168	0.9712	909.06
	35(40%)	Yx = 12,996 + 6.086X	23,510	0.9694	1,044.03
	42(50%)	Yx = 8,833 + 6.262X	23,486	0.9564	878.27

REGRESSION & CORRELATION STATISTICS (Continued)

Properties	No.	Equation	$\bar{X}$	R^2	F-Value	
Frame, 5 years old or newer, $313 to $412	56(60%)	Yx = 13,066 + 6.079X	24,547	0.9480	983.57	
	67(70%)	Yx = 14,887 + 5.994X	24,911	0.9371	968.57	
	78(80%)	Yx = 13,298 + 6.041X	25,800	0.9383	1,156.06	
	82(90%)	Yx = 13,169 + 6.054X	25,820	0.9390	1,230.68	
	96(100%)	Yx = 8,435 + 6.284X	26,882	0.9520	1,863.08	
Frame, more than 17 suites, $313 to $412	4(10%)	Yx =224,014 +-0.282X	32,502	0.0099	0.02	(18.51)
	8(20%)	Yx = 99,355 + 3.683X	32,868	0.3456	3.17	(5.99)
	14(30%)	Yx = 34,018 + 5.474X	33,593	0.9027	111.33	
	15(40%)	Yx = 36,315 + 5.443X	33,512	0.8888	103.93	
	18(50%)	Yx = 41,791 + 5.366X	33,160	0.8314	78.88	
	23(60%)	Yx = 65,067 + 4.764X	34,633	0.8538	122.62	
	29(70%)	Yx = 64,357 + 4.702X	34,247	0.7840	97.99	
	37(80%)	Yx = 50,487 + 5.085X	34,534	0.7943	135.18	
	39(90%)	Yx = 53,379 + 5.017X	34,609	0.7884	137.86	
	47(100%)	Yx = 22,442 + 5.972X	36,106	0.8732	309.98	
Frame, 5 years old or newer, 4 to 6 suites, Delton	6	Yx = 20,536 + 4.227X	8,285	0.7989	15.89	
Frame, 5 years old or newer, 4 to 6 suites, King Edward Park	4	Yx = -5,729 + 8.131X	8,153	0.5301	2.26	(18.51)

REGRESSION & CORRELATION STATISTICS (Continued)

Properties	No.	Equation	$\bar{X}$	R^2	F-Value	
Frame, 5 years old or newer, 4 to 6 suites, Hudson Bay Reserve	0	- - - -	- - - -	- - -	- - - -	
Frame, 5 years old or newer, 18 to 24 suites, Delton	5	Yx =-169,645 + 12.596X	28,964	0.6118	4.73	(10.13)
Frame, 5 years old or newer, 18 to 24 suites, King Edward Park	6	Yx = 151,691 + 2.193X	30,879	0.2232	1.15	(7.71)
Frame, 5 years old or newer, 18 to 24 suites, Hudson Bay Reserve	11	Yx = 56,085 + 4.973X	32,725	0.3615	5.10	(5.12)
Frame, 10 years old or newer, 18 to 24 suites, Delton	5	Yx =-169,645 + 12.596X	28,964	0.6118	4.73	(10.13)
Frame, 10 years old or newer, 18 to 24 suites, Hudson Bay Reserve	12	Yx = 40,525 + 5.431X	32,253	0.4755	9.07	
Frame, 10 years old or newer, 18 to 24 suites, $313 to $412	41	Yx = 24,301 + 5.912X	32,841	0.5314	44.22	

REGRESSION & CORRELATION STATISTICS (Continued)

Properties	No.	Equation	$\bar{X}$	R^2	F-Value	
Frame, 10 years old or newer, 18 to 24 suites, $363 and over	22	Yx = 55,180 + 5.130X	33,636	0.5690	26.41	
Frame, 10 years old or newer, 18 to 24 suites, Before Benson's white paper	19	Yx = 91,739 + 3.709X	30,950	0.1973	4.18	(4.45)
Frame, 10 years old or newer, 18 to 24 suites, After Benson's white paper	22	Yx = 35,603 + 5.615X	33,575	0.6034	30.43	
Frame, More than 17 suites, $363 and over, Before Benson's white paper	9	Yx = 53,752 + 5.119X	36,972	0.9161	76.39	
Frame, More than 17 suites, $363 and over, After Benson's white paper	16	Yx = 23,909 + 6.030X	41,276	0.9061	135.03	
Frame, 10 years old or newer, More than 17 suites, Hudson Bay Reserve, Before Benson's white paper	5	Yx = 42,892 + 5.433X	35,916	0.9138	31.80	

REGRESSION & CORRELATION STATISTICS (Continued)

Properties	No.	Equation	$\bar{X}$	R^2	F-Value	
Frame, 10 years old or newer, More than 17 suites, Hudson Bay Reserve, After Benson's white paper	8	Yx = 17,420 + 6.110X	32,690	0.6175	9.69	
Frame, More than 17 suites, $363 and over, Westmount-Glenora and Hudson Bay Reserve, Before Benson's white paper	4	Yx = 7,883 + 5.962X	43,720	0.9876	159.33	
Frame, More than 17 suites, $363 and over, Westmount-Glenora and Hudson Bay Reserve, After Benson's white paper	10	Yx = 55,952 + 5.285X	41,719	0.8269	38.22	
Frame, More than 17 suites, $363 and over, Parkallen and King Edward Park, Before Benson's white paper	3	Yx = 169,949 + 1.634X	31,644	0.0405	0.04	(161)
Frame, More than 17 suites, $363 and over, Parkallen and King Edward Park, After Benson's white paper	2	-----	----	----	-----	

REGRESSION & CORRELATION STATISTICS (Continued)

Frame, 5 years old or newer, $363 to $412	$N = 15$ $\bar{X} = 33,525$ $Yx = 10,604 + 6.397X$ $= 225,063$ $t_{N-2}Sy \cdot x = (2.160)(18,280)$ $= 39,485$ Confidence Interval = ±17.54% $R^2 = 0.9594$ F-value = 306.99
Frame, 5 years old or newer, 18 to 24 suites	$N = 15$ $\bar{X} = 33,294$ $Yx = 24,154 + 5.787X$ $= 216,826$ $t_{N-2}Sy \cdot x = (2.160)(23,900)$ $= 51,624$ Confidence Interval = ±23.81% $R^2 = 0.5173$ F-value = 13.93

PREDICTED VERSUS ACTUAL VALUES

Sample: 96 (Frame, 5 years old or newer, \$313 to \$412-- see Table 17)
Regression Equation: $Y_x = 8{,}435 + 6.284X$
$S_{y \cdot x} = 17{,}055$
$R^2 = 0.9520$

Predicted	Actual	Difference	% Difference
51,041	54,000	2,959	5.48
54,434	50,000	4,434	8.87
50,663	49,400	1,263	2.56
57,450	59,658	2,208	3.70
63,483	69,000	5,517	8.00
56,520	56,600	80	0.14
61,221	59,000	2,221	3.76
66,122	56,000	10,122	18.08
78,187	70,000	8,187	11.70
88,367	82,000	6,367	7.77
89,876	100,000	10,124	10.12
91,007	89,000	2,007	2.25
80,073	92,500	12,427	13.44
90,630	102,000	11,370	11.15
77,433	104,000	26,567	25.54
101,941	102,000	59	0.06
114,760	111,400	3,360	3.02
127,580	120,000	7,580	6.32
165,434	165,000	434	0.26
125,694	109,800	15,894	14.48
140,248	130,000	10,248	7.88

PREDICTED VERSUS ACTUAL VALUES (Continued)

Predicted	Actual	Difference	% Difference
135,875	144,853	8,978	6.20
137,760	123,500	14,260	11.55
100,433	101,000	567	0.56
126,071	130,000	3,929	3.02
135,372	130,000	5,372	4.13
144,169	151,500	7,330	4.84
137,684	150,000	12,316	8.21
147,186	139,000	8,186	5.89
147,186	144,000	3,186	2.21
135,120	127,000	8,120	6.39
160,382	153,000	7,382	4.82
159,251	141,600	17,651	12.47
164,907	164,000	907	0.55
155,858	159,000	3,142	1.98
161,890	156,000	5,890	3.78
150,956	119,500	31,456	26.32
158,120	144,500	13,620	9.43
130,219	124,000	6,219	5.02
166,792	160,000	6,792	4.24
166,792	148,000	18,792	12.70
160,005	154,500	5,505	3.56
161,513	160,000	1,513	0.95
153,595	151,000	2,595	1.72

PREDICTED VERSUS ACTUAL VALUES (Continued)

Predicted	Actual	Difference	% Difference
153,595	157,000	3,405	2.17
154,727	144,041	10,686	7.42
159,854	178,000	18,146	10.19
157,240	153,975	3,265	2.12
155,481	152,000	3,480	2.29
171,316	166,000	5,316	3.20
156,675	160,000	3,325	2.08
178,103	162,000	16,103	9.94
184,890	161,000	23,890	14.84
189,414	170,000	19,414	11.42
187,152	171,000	16,152	9.45
249,741	271,200	21,494	7.91
221,086	237,000	15,914	6.71
197,332	198,500	1,168	0.59
211,911	230,000	18,089	7.86
202,611	201,500	1,110	0.55
196,955	228,500	31,545	13.81
225,594	209,000	16,594	7.94
221,840	222,000	160	0.07
189,414	223,000	33,586	15.06
225,610	239,500	13,890	5.80
201,706	207,000	5,294	2.56
204,496	217,338	12,842	5.91
196,955	205,000	8,045	3.92

PREDICTED VERSUS ACTUAL VALUES (Continued)

Predicted	Actual	Difference	% Difference
219,577	228,000	8,423	3.69
196,653	233,500	36,847	15.78
215,807	225,000	9,193	4.09
229,757	229,300	457	0.20
209,523	227,000	17,477	7.70
202,416	213,000	10,584	4.97
214,550	209,000	5,550	2.66
232,774	186,000	46,774	25.15
212,037	195,000	17,037	8.74
232,773	185,700	47,073	25.35
221,211	220,000	1,211	0.55
217,692	203,000	14,692	7.24
224,856	238,500	13,644	5.72
212,791	229,100	16,309	7.12
224,479	248,000	23,521	9.48
227,118	241,000	13,882	5.76
209,774	215,000	5,226	2.43
210,378	248,800	38,422	15.44
246,599	265,000	18,401	6.94
262,786	274,100	11,314	4.13
257,281	247,500	9,781	3.95
272,363	260,000	12,363	4.76
340,230	341,725	1,495	0.44
408,097	353,100	54,997	15.58

PREDICTED VERSUS ACTUAL VALUES (Continued)

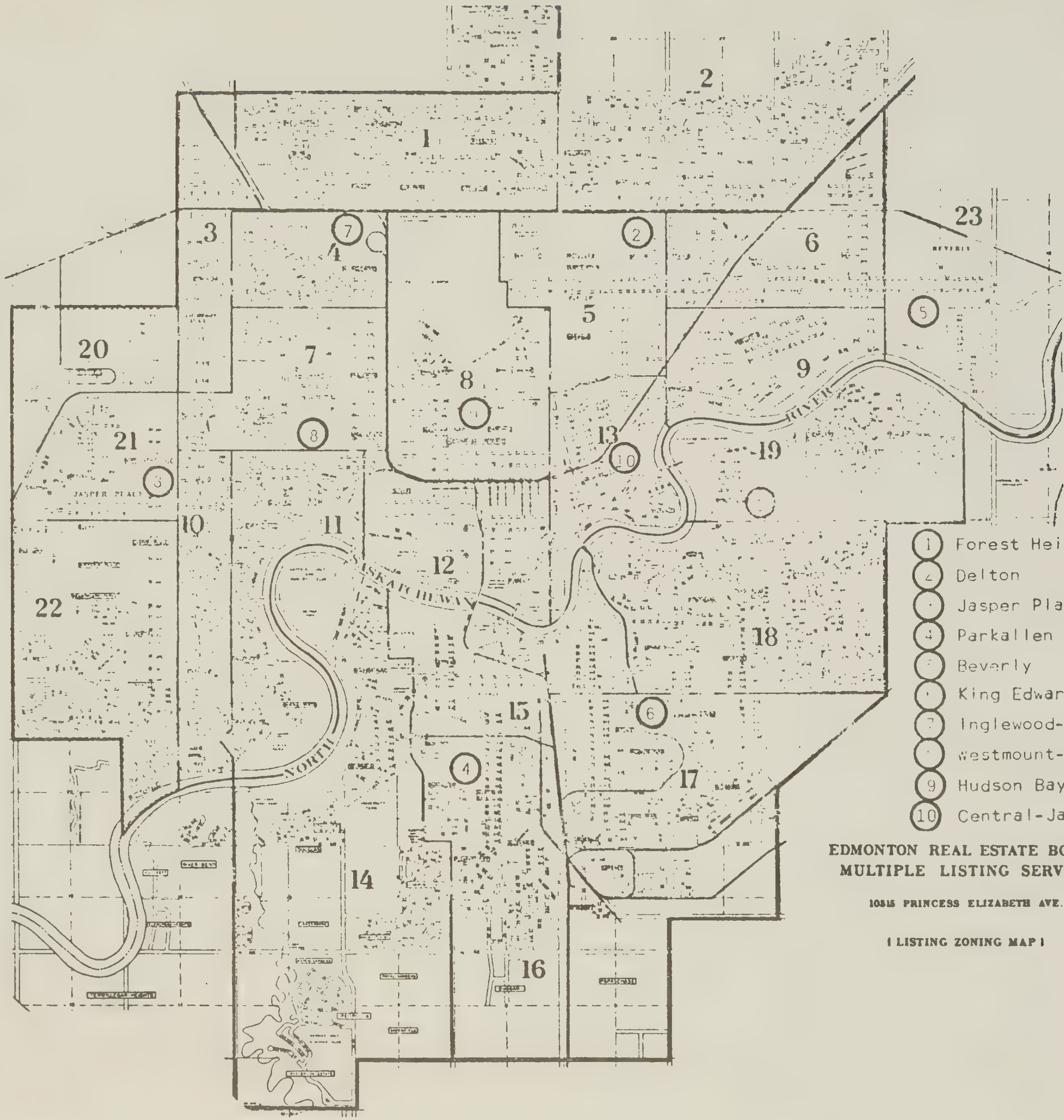
Predicted	Actual	Difference	% Difference
348,148	339,225	8,923	2.63
346,640	397,000	50,360	12.69
357,951	331,000	26,951	8.14
452,199	471,000	18,801	3.99
	Average % Difference		7.13

COMPUTER PROGRAM
(INITIAL RUN)

```
RUN NAME        STUDY OF GROSS RENT MULT RELIAB IN INC PROP APPRSL
FILE NAME       LITEPLO,THESIS DATA
VARIABLE LIST   PROP,PRICE,RENT,CONS,AGE,TIME,DATE,LOC,STES,ROOMS,AREA,DIST,INCRM
INPUT FORMAT    FIXED (F3.0,1X,F6.0,1X,F5.0,1X,F1.0,1X,F2.0,1X,F1.0,1X,F1.0,
                1X,F2.0,1X,F2.0,1X,F3.0,1X,F1.0,1X,F2.0,1X,F1.0)
# OF CASES      157
INPUT MEDIUM    CARD
VAR LABELS      PROP,IDENTIFICATION NUMBER OF PROPERTY/
                PRICE,SELLING PRICE OF PROPERTY/
                RENT,GROSS ANNUAL RENT AT DATE OF SALE/
                CONS,CONSTRUCTION OF PROPERTY/
                AGE,AGE OF PROPERTY IMPROVEMENTS AT DATE OF SALE/
                TIME,BEFORE OR AFTER WHITE PAPER/
                DATE,DATE OF SALE OF PROPERTY/
                LOC,LOCATION OF PROPERTY BY SUBDIVISION/
                STES,NUMBER OF SUITES/
                ROOMS,NUMBER OF ROOMS/
                AREA,QUARTER OF CITY/
                DIST,DISTRICT OF CITY BY GROUPING OF SUBDIVISIONS/
                INCRM,GROSS INCOME PER ROOM/
VALUE LABELS    CONS (1)MASONRY (2)FRAME/
                TIME (1)BEFORE NOV 9 1969 (2)AFTER NOV 9 1969/
                DATE (1)LAST HALF 1968 (2)FIRST HALF 1969 (3)LAST HALF 1969
                (4)FIRST HALF 1970 (5)LAST HALF 1970/
                AREA (1)NE QUARTER (2)NW QUARTER (3)SE QUARTER (4)SW QUARTER/
                DIST (1)FOREST HEIGHTS (2)DELTON (3)JAS PLACE (4)PARKALLEN
                (5)BEVERLY (6)K E PARK (7)INGLEWOOD-CALDER (8)WESTMOUNT-GLENORA
                (9)H B R (10)CENTRAL-JAS AVE/
                INCRM (1)312 AND LESS (2)313 TO 362 (3)363 TO 412 (4)413 AND OVER
REGRESSION      VARIABLES=PRICE,RENT/
                REGRESSION=PRICE WITH RENT(1)
STATISTICS      ALL
READ INPUT DATA
(157 CASES HERE)
FINISH
```

COMPUTER OUTPUT
(INITIAL RUN)

```
VARIABLE                 MEAN                 STANDARD DEV      CASES

PRICE              161198.3822                  91128.9290        157
RENT                23760.9299                  13517.2062        157

CORRELATION COEFFICIENTS..

                          PRICE          RENT

PRICE                   1.00000       0.97307
RENT                    0.97307       1.00000

*  *  *  *  *  *  *  *  * . *  *  M U L T I P L E   R E G R E S S I O N  *  *  *  *  *  *  *  *  *  *  *  *  *

DEPENDENT VARIABLE..      PRICE     SELLING PRICE OF PROPERTY

VARIABLE(S) ENTERED ON STEP NUMBER  1..   RENT      GROSS ANNUAL RENT OF PROPERTY

MULTIPLE R.          0.97307          ANALYSIS OF VARIANCE     DF      SUM OF SQUARES        MEAN SQUARE            F
R SQUARE             0.94687          REGRESSION                 1.1226671330311.55000122671330311.55000      2762.45958
STANDARD ERROR   21072.50384          RESIDUAL                 155.  68827814801.51920     444050418.07432

-------------------- VARIABLES IN THE EQUATION --------------------       ----------- VARIABLES NOT IN THE EQUATION ----------

VARIABLE                B          BETA      STD ERROR B        F             VARIABLE    BETA IN    PARTIAL TOLERANCE        F

RENT                  6.56017    0.97307     0.12482        2762.460
 (CONSTANT)        5322.72267

MAXIMUM STEP REACHED


DEPENDENT VARIABLE..      PRICE     SELLING PRICE OF PROPERTY

                                                 SUMMARY TABLE

VARIABLE                                   MULTIPLE R   R SQUARE   RSQ CHANGE   SIMPLE R           B            BETA

RENT    GROSS ANNUAL RENT OF PROPERTY        0.97307     0.94687     0.94687    0.97307        6.56017        0.97307
 (CONSTANT)                                                                                 5322.72267
```



1 Forest Heights
2 Delton
3 Jasper Place
4 Parkallen
5 Beverly
6 King Edward Park
7 Inglewood-Calder
8 Westmount-Glenora
9 Hudson Bay Reserve
10 Central-Jasper Avenue

EDMONTON REAL ESTATE BOARD
MULTIPLE LISTING SERVICE

10815 PRINCESS ELIZABETH AVE.

(LISTING ZONING MAP)

B30014